修 士 論 文

原子力発電所 PRA のパラメータ推定方法を用いた
防火設備の信頼性分析

平 成 ２８ 年 度

２０１６年９月

東京理科大学大学院

国際火災科学研究科 火災科学専攻

沈 彦虹

Master's Thesis

RELIABILITY ANALYSIS OF FIRE SAFETY APPARATUS USING　PARAMETER ESTIMATION METHODS FOR PRA IN NUCLEAR POWER PLANT

September 2016

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Department of Fire Science and Technology

Graduate School of Global Fire Science and Technology

TOKYO UNIVERSITY OF SCIENCE

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1   Introduction

On March 22, 1975, a fire occurred at the Browns Ferry Nuclear Power Plant (BFN), operating near Decatur, Ala., which was caused by a candle flame that used for air leakage test through a non-fire-rated (polyurethane foam) penetration seal has fundamentally changed how the NRC dealt with fire protection at U.S. nuclear power plants.

In chapter 2, I will make a brief introduction of the Brown Ferry Fire how the fire was initiated and propagated, and the following firefighting. This accident revealed shortcomings both in fire protection design at nuclear power plants and the licensee's procedures for responding to a fire, which makes the NRC improve the fire protection documents and develop a new approach for managing fire safety.

The improved the fire protection documents and the new developed approach for managing fire safety called risk-informed, performance-based fire protection due to the BFN will be stated in chapter 3. In this chapter, the regulatory changes in fire protection after the BFN will be introduced in chronological order. Meanwhile, the regulatory changes in corresponding to two approach respectively are also described.

For the new fire protection approach, risk-informed, performance-based, the Probabilistic Risk Assessment (PRA) is typically used for estimating risk by computing real numbers to determine what can go wrong, how likely is it, and what are its consequences. The knowledge of general PRA and the common structure for Fire PRA will be introduced in Chapter 4. In addition, four terms used in fire protection and the development of their definitions due to the improvement of fire protection requirements are also stated in this chapter.

The knowledge in previous chapters could be considered as the foreshadowing for the Chapter 5. Since no quantitative analysis is carried out for fire safety apparatus in common building, I would like to utilize the parameter estimation for PRA in nuclear power plant for estimating the failure rates of the emergency lighting equipment (one kind of fire safety apparatus) and quantify the uncertainties in the estimates. The analysis procedures and the analysis results are stated in this chapter.

* The word or phrase in each chapter with a subscript [A] has a description in corresponding Appendix.

2   Browns Ferry Nuclear Fire (BNF)

2.1   Browns ferry design and layout

The cable spreading room is located directly below the control room. The Unit 1 reactor building is adjacent to the cable spreading room. Electrical cables from equipment throughout the plant converge in the cable spreading room before passing up to the control room shown in Figure 2-3 [2]. Figure 2-1 [1] shows the inside structure of the browns ferry plant. Figure 2-2 simplifies the structure and show the cable path as well.
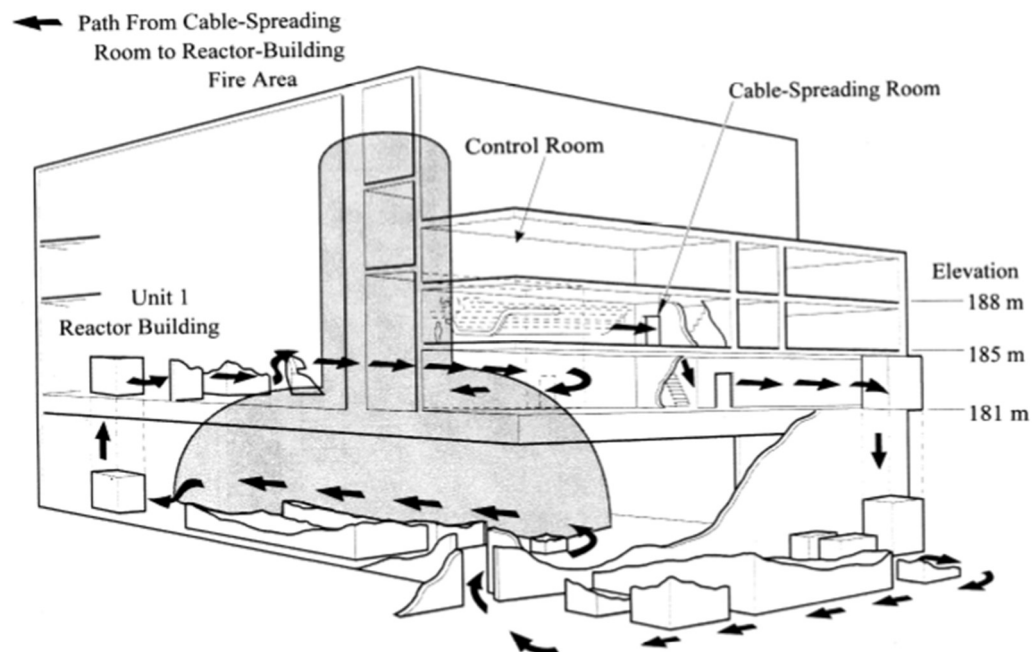
Figure 2-1   Inside structure of the browns ferry plant
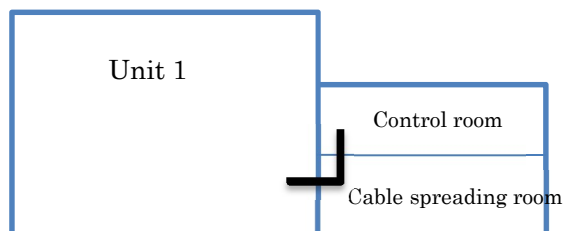
Figure 2-2   Simplified structure of Brown ferry plant with cable path

The walls, floors and ceiling of the cable spreading room are pierced by trays carrying thousands of cables. Figure 2-4 shows some of the cable trays and cables inside the cable spreading room. [2]

Figure 2-3　Control room　　　　Figure 2-4　Cables in cable spreading room

Electrical cables carry signals from sensors to chart recorders, gauges, and computers in control room to monitor plant conditions. Other cables carry signals from switches, dials, and push-buttons in the control room to valves, pumps, fans, and other plant equipment managing their operation. [2]

The penetrations were sealed to prevent potentially radioactive air leaking in the cable spreading room and then rising up into the control room. A slight pressure difference is maintained (by design) across this wall, with the higher pressure being on the cable spreading room side. Figure 2-5 shows the polyurethane foam used to fill a cable tray penetration. [4]
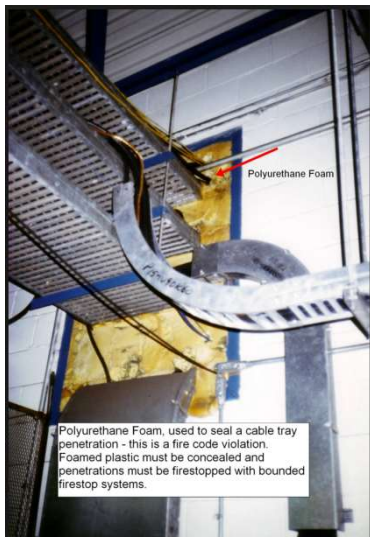
Figure 2-5　Polyurethane foam used to fill a cable tray penetration at a power plant

2.2　Process of fire [3][5]

The way for penetration seal testing was quick and easy. The worker held the candle as close to the wall penetrations as possible. A still flame meant no leakage while a

flickering candle flame indicated air leakage.

On that day, the cable tray penetration was leaking badly, so the air rushing past the candle fanned its flame. The flame ignited the combustible material, polyurethane foam, which was used to seal gaps (This is a fire code violation. Foamed plastic must be concealed and penetrations must be fire stopped with bounded firestop systems). The fire quickly spread and burned out of control.

2.3 Fire detection

There were smoke detectors in control room and cable spreading room, but no detector installed in Unit 1 reactor building [5].

The smoke detectors in both cable spreading room and control room didn't alarm. The reasons are as follows,
> In the cable spreading room, the normal flow of air from the spreading room to the reactor building drew the smoke away from the detectors.
> In the control room, the detectors which were of the ionization type didn't detect the products of combustion generated by the cable fire. Because ionization detectors lack mobility in a static electric field, they may not detect large smoke particles [6].

2.4 Fire fighting

The sequence of events caused by the fire is listed in table 2-1.

Table 2-1   Timeline of the sequence of events [5]

12:20	Fire occurred
12:20-12:35	Tried to put out the flame using a flashlight and rags, and then use CO_2 extinguishers
12:45-13:00	CO_2 fire system actuated—could not be immediately activated because the power had been cut off and a metal plate had been placed over CO2 controllers.
13:09	the workers notified the control room to call the fire department

	for support.
13:09-16:30	CO_2 and dry chemical extinguishers were being used to put out fire
17:30-18:00 (six hours later)	The water was authorized to be used. (the local fire chief suggested using water to fight the fire, but the plant manager refused.)
19:00	The use of fire hoses and water was starting.
19:45	It was confirmed that fire was extinguished.

2.5　Consequences of fire [5]

In the cable spreading room,
- ➢ The fire was controlled
- ➢ The damage was limited to about 5 feet(1.524m)

Because of the installed carbon dioxide extinguishing system and manual firefighting efforts

In the Unit1 reactor building
- ➢ The damage area is roughly 40 feet by 20 feet(12.192m*6.096m)
- ➢ About 1600 cables were damaged.

Much of the installed equipment lost control power because the electrical cables shorted after insulation had been burned off. All of the emergency core cooling systems for Unit 1 were rendered inoperable, and portions of the Unit 2 systems were likewise affected.

However, sufficient equipment to shut down the reactors and maintain the reactor cores in a cooled and safe condition remained operational. Therefore, there is no release of radioactive material above the levels associated with normal plant operation.

In addition to the cable damage, a dense soot created by the burning insulation was deposited throughout the Unit1 reactor building and in same small areas in the Unit 2 reactor building.

In conclusion, the fire, although limited principally to a 20*40 interior space,

- Caused extensive damage to electric power and control systems,
- Impeded the functioning of normal and standby cooling systems,
- Degraded the capacity to monitor the status of the plant,
- and caused both units to be out of service for many months.

2.6　Analysis after fire [5]

The effectiveness of redundancy depends on the independence of the redundant equipment. However, the Browns Ferry fire negated the redundancy. The two errors causing the independence as follows:

1. Wires connecting indicator lamps in the control room to control circuits for redundant safety equipment were not separated from each other. —fire damaged some of these wires causing the unavailability of the redundant equipment.

2. Wires of redundant subsystems were routed in the same area in the mistake belief that putting one set of such wires in electrical conduit would protect it. —the conduit got too hot and the wires in it short-circuited.

In order to maintain adequately effective independence of redundant safety equipment, a suitable combination of electrical isolation, physical distance, barriers, resistance to combustion, and sprinkler systems should be applied. It indicated the importance of the compartmentation of the building which is stated in chapter 4.

There is also another lesson learned from this fire that the importance of immediately notifying the control room once detecting a fire. In this case, control room operators will dispatch firefighters in time and take appropriate compensation measures. [2]

2.7   Supplement

Table 2-2   Conditions of the reactors complying with the fire regulations [3]

1980	the NRC adopted new fire regulations (10CFR50.48, Appendix R of 10 CFR50) --required plants to physically separate electrical cables for a primary safety system from the cables for its backup, or to heavily insulate the cables
2000(two decades later)	Many reactors in the U.S. had still not complied with its 1980 fire protection regulations
2004	The NRC revised its fire protection regulations (10 CFR 50.48(c))—relied on NFPA 805. Allowed electrical cables to be side-by-side.
2009.3.4	The TVA, Browns Ferry's owner, promised the NRC that it would transition to the 2004 regulations and submit that plan to the NRC by March 4, 2012.
2012.1.13	TVA wrote to the NRC asking to postpone the deadline until March 29, 2013
2012.5.18	The NRC granted the request by the TVA
2012	Only 4 reactors have complied. Today (2012.6.26), 47 of the 51 reactors still don't comply with either the 1980 or the 2004 fire regulations. The three reactors at Browns Ferry are among those that fail to comply.

* the fire protection regulations stated here will be introduced in Chapter 3

2.8   Reference

[1]   U.S. Nuclear Regulatory Commission. (1975, March 22). *Browns Ferry Fire DRP Heritage Discussion.* Retrieved from http://www.nrc.gov/reading-rm/doc-collections/nuregs/brochures/br0361/s1/bfn-fire-heritage.pdf

[2]   Lochbaum, D. (2010, October 19). *Fission Stories #16: Candle in the Wind.* Retrieved from Union of Concerned Scientists Science for a healthy planet and safer world: http://allthingsnuclear.org/dlochbaum/fission-stories-16-candle-in-the-wind

[3]   Lochbaum, D. (2012, June 26). *Fission Stories #98: Fires at Browns Ferry: Get Your Fiddles Ready.* Retrieved from Union of Concerned Scientists Science for a healthy planet and safer world: http://allthingsnuclear.org/dlochbaum/fission-stories-98-fires-at-browns-ferry-get-your

[4]   Browns Ferry Nuclear Power Plant. (2016, June 23). In *Wikipedia, The Free Encyclopedia.* Retrieved 02:43, July 24, 2016, from https://en.wikipedia.org/w/index.php?title=Browns_Ferry_Nuclear_Power_Plant&oldid=726719167

[5]   U.S. Nuclear Regulatory Commission. (1976). Recommendations related to Browns Ferry fire. *NUREG-0050.*

[6]   [6] Berry, D. L. (March 1979). *Nuclear Power Plant Fire Protection Fire Detection (Subsystems Study Task 2), NUREG/CR-0488.* Sandia National Laboratories Albuquerque, NM 87185. pp 19-20

3 Development of Fire Protection in Nuclear Power Plant

3.1 Background on Fire Protection for Nuclear Power Plants

On March 22, 1975, a fire at the Browns Ferry Nuclear Power Plant (BFN), operating near Decatur, Ala., fundamentally changed how the NRC dealt with fire protection at U.S. nuclear power plants. The fire started when plant workers in the cable spreading room used a candle flame to check for air leakage through a non-fire-rated (polyurethane foam) penetration seal that led to the reactor building. The fire ignited both the seal material and the cables passing through it.

It took almost seven hours to extinguish it. More than 1600 cables were affected, and 628 of which were important to plant safety. The fire damage to the cables for electrical power, control systems, and instrumentation affected the function of both normal and standby reactor cooling systems and the operators' plant monitoring capability. In this case, the operators had to initiate emergency repairs to restore the systems needed to shut the reactor down safely. **(As stated in chapter 2)**

Investigations after the fire revealed shortcomings both in fire protection design at nuclear power plants and the licensee's procedures for responding to a fire. [1] Therefore, the Nuclear Regulatory Commission (NRC) improved the fire protection regulations and developed something new approach for managing fire safety.

3.2   Fire protection regulations

Referring to the "Backgrounder on Fire Protection for Nuclear Power Plants" [1] published by the U.S. NRC and then searching the relevant documents, I pick the main fire protection documents developed after the Browns Ferry Fire occurring and make some brief introduction as follows. (introducing in time order)

3.2.1   BTP APCSB 9.5-1(May, 1976)

--"Guidelines for Fire Protection for Nuclear Power Plants"

After the investigations of the Browns Ferry Fire, the NRC Browns Ferry special review team recommended the NRC to develop detailed guidance for implementing the general design criterion for fire protection and to conduct a detailed review of the fire protection program at each operating nuclear power plant, comparing it to the guidance developed.[2]

In May 1976, the NRC developed the Branch Technical Position (BTP) Auxiliary Power Conversion System Branch (APCSB) 9.5-1 which incorporated those recommendations from the NRC Brown Ferry special review team to describe guidelines acceptable for implementing the **General Design Criterion 3 (GDC 3) of Appendix A** to 10 CFR Part 50 for nuclear power plants. **[3]** However, the guidelines of APCSB 9.5-1 applied only to those plants licensed after July 1, 1976. In this case, in September 1976, the NRC modified the guidelines in APCSB 9.5-1 and issued Appendix A to APCSB 9.5-1 which could apply to the plants licensed prior to July 1, 1976. [2]

   * The guidelines for fire protection are listed in **Table 3-1.**

Table 3-1 Guidelines for Fire Protection in Nuclear Power Plants [4]

Document	Document Title
SRP 9.5.1 (Section 9.5.1 of NUREG-0800)	Section 9.5.1, "Fire Protection Program" of NUREG-0800, "Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants, LWR Edition," various dates and revisions
APCSB 9.5-1	Branch Technical Position APCSB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants" May 1, 1976
ASB 9.5-l,	Branch Technical Position ASB 9.5-1 "Guidelines for Fire Protection for Nuclear Power Plants," Revision 1, March 1979.
CMEB 9.5-1 (Formerly ASB 9.5-1)	Branch Technical Position CMEB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants" Revision 2, July 1981.
SPLB 9.5-1 (Formerly CMEB 9.5-1)	Branch Technical Position SPLB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants" Revision 4, October 2003.

3.2.2 Section 50.48 and Appendix R to 10 CFR 50 (November, 1980)

[10 CFR 50(Title 10 of the Code of Federal Regulations)— "Domestic Licensing of Production and Utilization Facilities"]

In November 1980, a new section 50.48 and Appendix R were added to the 10 CFR 50. (Became effective on February 17, 1981) [2]

3.2.2.1　10 CFR 50.48

− "Fire protection"

The fire vision of section 50.48 is read as follows [5] and then quickly replaced.

> **§ 50.48　Fire protection schedules.**
> To the extent that any facility's license conditions or technical ipecifications incorporate compliance lates for modifications necessary to rovide fire protection features proposed by a licensee and accepted by the NRC staff as satisfying the provisions of Appendix A to Branch Technical Position BTP/APCSB 9.5–1 and reflected in NRC staff Fire Protection Safety Evaluation Reports issued prior to the effective date of this rule, those dates are hereby suspended pending further action by the Commission.

This section including section (a) and (b) requires that each operating nuclear power plant have to satisfy General Design Criterion 3 of Appendix A to 10 CFR Part 50, and also requires that all plants operating prior to January 1, 1979 satisfy the requirements of Appendix R to 10 CFR Part 50 with some exception.

3.2.2.2　Appendix R to 10 CFR 50

--"Fire Protection Program for Nuclear Power Facilities Operating Prior to January 1, 1979"

The Appendix R known as "Fire Protection Program for Nuclear Power Facilities Operating **Prior to** January 1, 1979" applied to nuclear power plants licensed to operate before January 1, 1979, with exception that stated in **§ 50.48(b)** of this part. With respect to certain generic issues for these plants, it established fire protection features required to satisfy General Design Criterion 3 of Appendix A to this part.

3.2.3　Regulation Guide 1.189 (April 2001; October 2009)

--"Fire Protection for Nuclear Power Plants"

The NRC developed the Regulation Guide 1.189 to provide a comprehensive fire protection guidance document and to identify the scope and depth of fire protection that the staff would consider acceptable for nuclear plants operating by January 1, 2001. This guide may be used for licensee self-assessments and as the deterministic basis for future rulemaking.[2] Many sections of this guidance are based on CMEB9.5-1.

The revision published in 2009 provides guidance for new reactor designs (after January 1, 2001). Besides, the revision incorporates the guidance previously included in Branch Technical Position **(BTP) SPLB 9.5-1**, "Guidelines for Fire Protection for Nuclear Power Plants" (formerly BTP CMEB 9.5-1), issued October 2003.[6]

3.2.4　NFPA 805 (January 13, 2001)

--"Performance-Based Standard for Fire Protection for Light Water Reactor Electric Generating Plants"

On January 13, 2001, the National Fire Protection Association (NFPA) Standards Council approved NFPA Standard 805 as a risk-informed, performance-based standard for existing light-water nuclear power plants. The staff of the U.S. Nuclear Regulatory Commission (NRC) also participated in the development of NFPA 805 which published

in February 2001.

The developed NFPA 805 describes a methodology for existing light-water nuclear power plants to apply risk-informed, performance-based requirements and fundamental fire protection design elements to establish fire protection systems and features required for all modes of reactor operation. In addition, it presents a methodology for establishing fire protection procedures, systems, and features for nuclear power plants that are decommissioning and permanently shut down. [7]

3.2.5 10 CFR 50.48 (c) (July 16, 2004)

(-- Title 10, Part 50, Section 48(c) of the Code of Federal Regulations)

On June 16, 2004, the NRC published the **10 CFR 50.48 (c)** endorsing the NFPA 805 with some exceptions as an alternative to the traditional deterministic fire protection requirements for the reactor licensees complying with. It became effective on July 16, 2004.

3.2.6 NEI 04-02 (September 2005)

(-- Guidance for Implementing a Risk-informed, Performance-based Fire Protection Program under 10 CFR 50.48(c))

The NEI 04-02 documented by Nuclear Energy Institute (NEI) provided guidance for implementing the requirements of the rule change stated above. This document, to the extent endorsed by the NRC, also represented methods acceptable to the NRC for implementing in whole or in part a risk-informed, performance-based fire protection program. [8]

3.2.7 Regulation Guide 1.205 (June 2006; September 2009)

(-- Risk-informed, Performance-based Fire Protection for Existing Light-water Nuclear Power Plants)

This regulatory guide provides guidance for complying with the requirements for

risk-informed, performance-based fire protection programs (FPPs) approved by the NRC that comply with 10 CFR 50.48(c) and the referenced 2001 Edition of the NFPA 805. [9]

As noted before, the NEI developed NEI 04-02 to assist licensees in adopting 10 CFR 50.48(c) and making the transition from their current FPP to one based on NFPA 805. This regulatory guide endorses portions of NEI 04-02. However, the regulatory positions in Section C include clarification of the guidance provided in NEI 04-02, as well as NRC exceptions to the guidance. So the regulatory positions in Section C take precedence over the guidance in NEI 04-02. [9] Nevertheless, the NRC also endorses the NEI 04-02, because its methods acceptably implement NFPA 805 and comply with the Regulation Guide 1.205. [1]

Those regulatory documents referred above are shown in **Figure 3-1**.
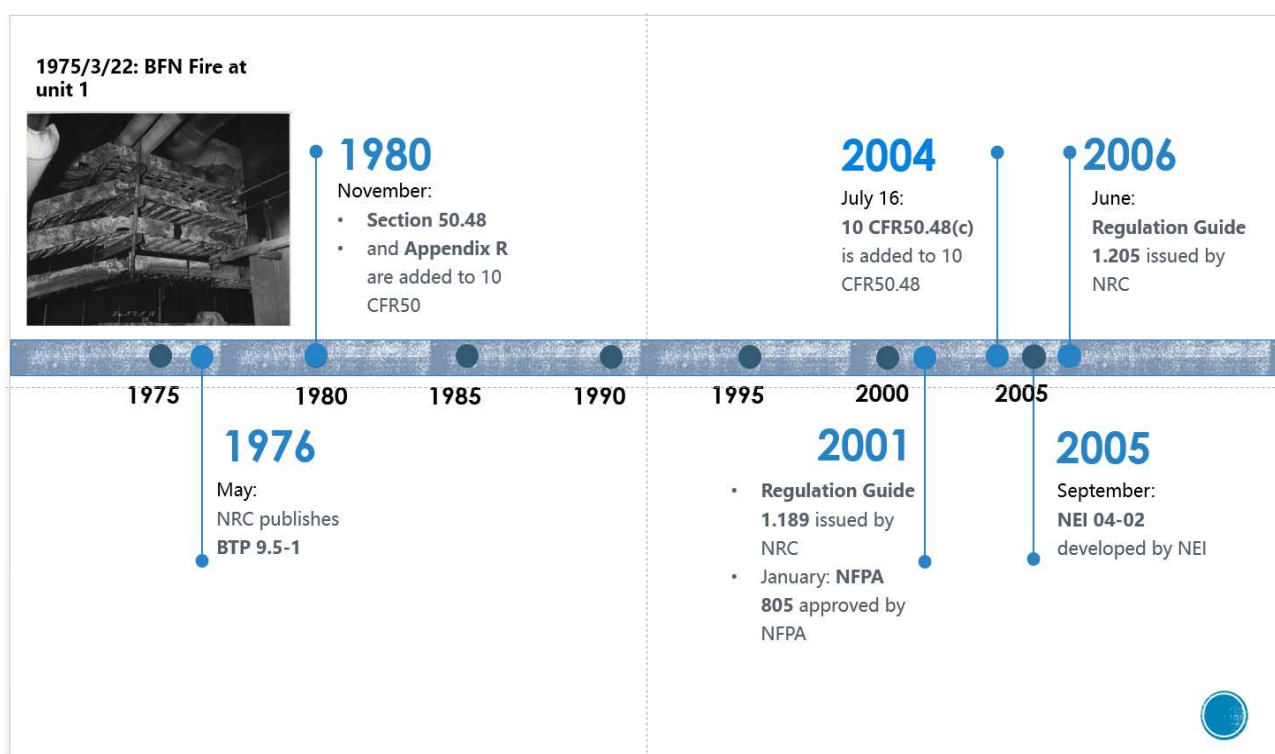
Figure 3-1 Timeline of the regulatory changes in fire protection due to Browns Ferry Nuclear (BFN) Fire

3.3 Two approaches for fire protection

Nowadays, plants can choose between two approaches for managing their fire safety:

- Deterministic fire protection

The deterministic fire protection establishes fire safety by ensuring systems needed to shut down the reactor will survive a fire. This approach, based on an assumed serious fire, was developed when the staff and the industry had system-based tools for considering fire risk. [1]

Plants that were licensed before January 1, 1979 were also subject to the prescriptive requirements of 10 CFR 50.48(b) and Appendix R of 10 CFR 50. Plants that were licensed after January 1, 1979 also followed the same prescriptive requirements to conform to the fire protection regulations. [10]

- Risk-informed, performance-based fire protection

The risk-informed performance-based fire protection approach considers risk insights as well as other factors to better focus attention and resources on design and operational issues according to their importance in safety. This approach relies on a required outcome rather than requiring a specific process or technique to achieve that outcome.

The deterministic approach involved asking only what can go wrong and what the consequences are. However, the risk-informed performance-based approach asked an additional question of how likely it is that something will go wrong. Many of the present regulations are based on deterministic requirements and they are cannot be quickly replaced by the risk-informed performance-based regulations. Thus, the traditional deterministic regulations are being maintained, while risk-informed performance-based regulations are being developed and implemented. [11]

3.4 Fire protection regulations for two approaches

Among the fire protection documents, the 10 CFR 50.48(b), Appendix R of 10 CFR 50 and Regulatory Guide 1.189 can be applied to the deterministic fire protection, on the other hand, 10 CFR 50.48(c), NFPA 805, NEI 04-02 and Regulatory Guide 1.205 can be applied to the risk-informed, performance-based fire protection. Moreover, plants that

have adopted either a deterministic FPP or a performance-based FPP must meet the 10 CFR 50.48 (a). The documents applied to those two approaches are listed in in **Table 3-2**

Table 3-2   The documents applied to the two fire protection approaches

Documents	Published times
Deterministic fire protection	
10 CFR 50.48[(a),(b)]	November 1980
Appendix R of 10 CFR 50	November 1980
Regulatory Guide 1.189	April 2001; October 2009
Risk-informed, performance-based fire protection	
10 CFR 50.48(a)	November 1980
10 CFR 50.48(c)	July 16, 2004
NFPA 805	January 13, 2001
NEI 04-02	September 2005
Regulatory Guide 1.205	June 2006; September 2009

For **Deterministic fire protection**

The deterministic fire protection is the traditional fire protection approach used by the NRC to ensure the safety of U.S. nuclear power plants. In order to comply with this fire protection requirements, every plant must have a fire protection plan that satisfies 10 CFR 50.48(a) and 10 CFR Part 50, Appendix A, Criterion 3 outlining [10]:

(1) The fire protection program,

(2) Installed fire protection systems,

(3) And the means to assure the reactor can be safely shutdown in the event of a fire.

The NRC lists deterministic requirements in 10 CFR 50.48(b) and Appendix R of 10 CFR 50. Regulatory Guide 1.189 provides guidance to plants for meeting these requirements. These relationships are shown in **Figure 3-2.**

□Deterministic Fire Protection
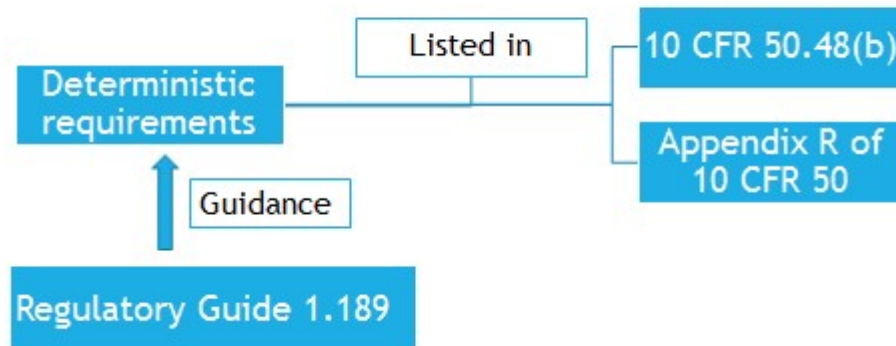
Figure 3-2 documents for deterministic fire protection

For **risk-informed, performance-based fire protection**

- NFPA 805 describes a methodology for existing light-water nuclear power plants to apply risk-informed, performance-based requirements.

- The NRC approved incorporating NFPA 805 into Section c of the Code of Federal Regulations 10 CFR Part 50.48 [10 CFR Part 50.48(c)], with some exceptions.

- The Nuclear Energy Institute (NEI) has developed NEI 04-02 to assist licensees in adopting 10 CFR 50.48(c) and making the transition from their current fire protection program (FPP) to one based on NFPA 805.

- Regulatory guide 1.205 also provides guidance for use in complying with the requirements of 10 CFR 50.48(c) and NFPA 805, endorsing NEI 04-02.

Figure 3-3 Documents related to the Alternate Fire Protection Rule [National Fire Protection Association (NFPA) Standard 805] [12]

3.5　Appendix

A. U.S. Nuclear Regulatory Commission Documents

Regulations

10 CFR Part 50, "Domestic Licensing of Production and Utilization Facilities"
　　10 CFR 50.48, "Fire Protection."
　　GDC 3, "Fire Protection," of Appendix A, "General Design Criteria for Nuclear Power Plants," to 10 CFR Part 50.
　　Appendix R, "Fire Protection Program for Nuclear Power Facilities Operating Prior to January 1, 1979," to 10 CFR Part 50.

Regulatory Guides
Regulatory Guide 1.189, "Fire Protection for Nuclear Power Plants"

Regulatory Guide 1.205, "Risk-informed, Performance-based Fire Protection for Existing Light-water Nuclear Power Plants"

NUREG-Series Reports
NUREG-0800, "Standard Review Plan for the Review of Safety Analysis Reports for Nuclear Power Plants (LWR Edition)," Section 9.5-1, "Fire Protection Program" **(SRP 9.5.1)**

Branch Technical Positions (BTP)
APCSB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants," May 1, 1976.

Appendix A to APCSB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants, Docketed Prior to July 1, 1976," February 24, 1977.

ASB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants," Revision 1, March 1979.

CMEB 9.5-1 (Formerly ASB 9.5-1), "Guidelines for Fire Protection for Nuclear Power Plants," Revision 2, July 1981.

SPLB 9.5-1 (Formerly CMEB 9.5-1), "Guidelines for Fire Protection for Nuclear Power

Plants," Revision 4, October 2003.

National Fire Protection Association (NFPA) Codes and Standards
NFPA 805, "Performance-based Standard for Fire Protection for Light Water Reactor Electric Generating Plants"

Nuclear Energy Institute (NEI)
NEI 04-02, "Guidance For Implementing A Risk-informed, Performance-based Fire Protection Program under 10 CFR 50.48(c)"

B. List of acronyms
BFN: Browns Ferry Nuclear

BTP: Branch Technical Position

CFR: Code of Federal Regulation

FPP: Fire Protection Program

GDC: General Design Criterion

NEI: Nuclear Energy Institute

NFPA: National Fire Protection Association

NPP: Nuclear Power Plant

NRC: U.S. Nuclear Regulatory Commission

PRA: Probabilistic Risk Assessment

RES: Office of Nuclear Regulatory Research

3.6   Reference

[1]   U.S. Nuclear Regulatory Commission. (2016, April 08). *Backgrounder on Fire Protection for Nuclear Power Plants.* Retrieved from http://www.nrc.gov/reading-rm/doc-collections/fact-sheets/fire-protection-fs.html

[2]   U.S. Nuclear Regulatory Commission. (April 2001). *Regulatory Guide-1.189 Fire Protection for Operating Nuclear Power Plants.*

[3]   U.S. Nuclear Regulatory Commission. (May 1, 1976). *BTP APCSB 9.5-1, Guidelines for Fire Protection for Nuclear Power Plants.* Washington, DC.

[4]   U.S. Nuclear Regulatory Commission. (2016, March 30). *Guidelines for Fire Protection in Nuclear Power Plants.* Retrieved from http://www.nrc.gov/about-nrc/fire-protection/related-info/guide-npp.html

[5]   U.S. Nuclear Regulatory Commission. (n.d.). *10 CFR PART 50.48: HISTORY OF CHANGES.*

[6]   U.S. Nuclear Regulatory Commission. (October 2009). *Regulatory Guide 1.189 Fire Protection for Nuclear Power Plants.* Washington, DC.

[7]   U.S. Nuclear Regulatory Commission. (2016, April 15). *Overview of the Alternate Fire Protection Rule [10 CFR 50.48(c)].* Retrieved from http://www.nrc.gov/reactors/operating/ops-experience/fire-protection/protection-rule/protection-rule-overview.html

[8]   Nuclear Energy Institute. (September 2005). *Guidance for Implementing a Risk-Informed, Performance-Based Fire Protection Program Under 10 CFR 50.48 (c), NEI 04-02, Revison 1.* Washington,DC.

[9]   U.S. Nuclear Regulatory Commission. (May 2006). *Risk-Informed, Performance-Based Fire Protection for Existing Light-Water Nuclear Power Plants, Regulatory Guide 1.205.*

[10]   U.S. Nuclear Regulatory Commission. (2016, April 15). *Deterministic Fire Protection.* Retrieved from http://www.nrc.gov/reactors/operating/ops-experience/fire-protection/deterministic.html

[11]   U.S. Nuclear Regulatory Commission. (2013, July 18). *History of the NRC's Risk-Informed Regulatory Programs*. Retrieved from http://www.nrc.gov/about-nrc/regulatory/risk-informed/history.html

[12]   U.S. Nuclear Regulatory Commission. (2014, February 25). *Alternate Fire Protection Rule [10 CFR 50.48(c), NFPA 805]*. Retrieved from http://www.nrc.gov/reactors/operating/ops-experience/fire-protection/protection-rule .html

4 Nuclear Power Plant Probabilistic Risk Assessment (PRA)

4.1 Background

The Nuclear Regulatory Commission's responsibilities include ensuring U.S. nuclear power plants and other licensed facilities operate with minimal risk to public health and safety. [1]

Initially, the NRC developed many of its regulations following the deterministic regulatory requirements without considering risk estimation.

Since the 1970s, the NRC and its licensees have used risk assessment in some areas. However, for reactors, the NRC did not systematically quantify the probabilities of accidents until 1975 when the agency published the Reactor Safety Study (WASH-1400, NUREG/75-014). Since 1975, the NRC and its licensees have advanced significantly in their knowledge of (and experience with) probabilistic risk assessment (PRA).

The traditional deterministic approach involved asking only what can go wrong and what the consequences are, however, PRA is required to ask the additional question of how likely it is that something will go wrong. [2]

4.2 Definition of PRA

Probabilistic Risk Assessment (PRA) is a systematic method for estimating risk by computing real numbers to determine what can go wrong, how likely is it, and what are its consequences. Thus, PRA provides insights into the strengths and weaknesses of the design and operation of a nuclear power plant. [3]

4.3 Types of risk assessments

Nuclear power plant PRAs deal with "internal events"—those that start inside the power plant or the electric system it serves—and "external event" such as earthquake, floods, and hurricanes.

For the type of nuclear plant currently operating in the United States, a PRA can estimate three levels of risk [3]. **Figure 4-1** shows the flow path of these three levels.

A Level 1 PRA estimates the frequency of accidents that cause damage to the nuclear reactor core. This is commonly called **core damage frequency (CDF)**[A].

A Level 2 PRA, which starts with the Level 1 core damage accidents, estimates the frequency of accidents that release radioactivity from the nuclear power plant.

A Level 3 PRA, which starts with the Level 2 radioactivity release accidents, estimates the consequences in terms of injury to the public and damage to the environment.
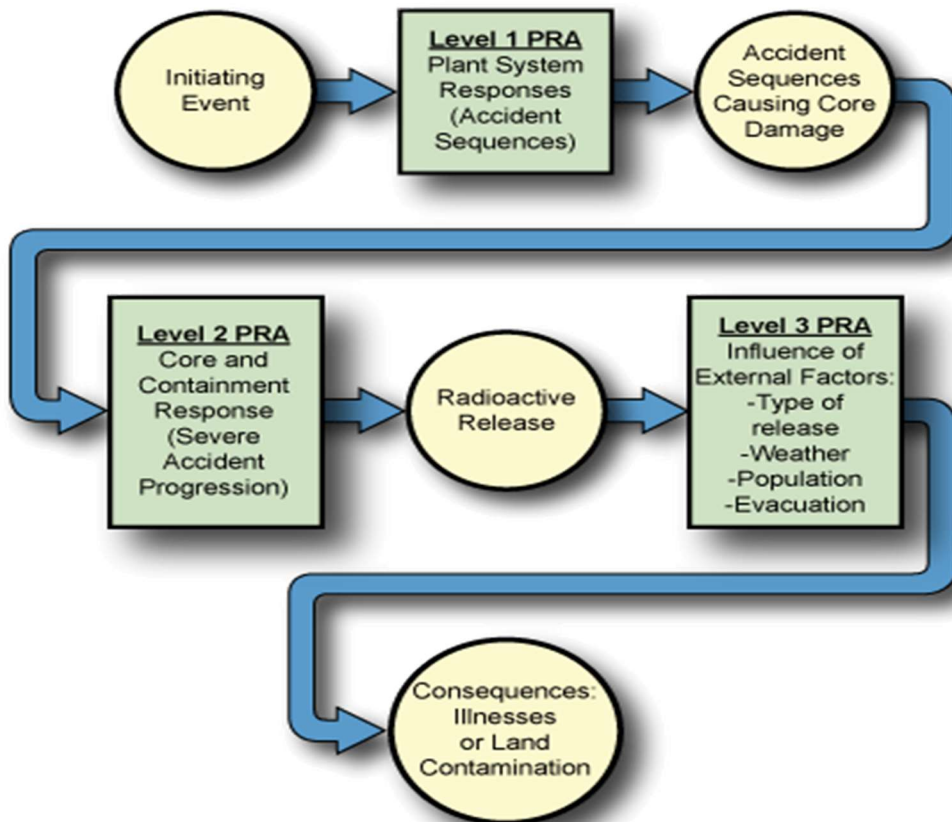
Figure 4-1　Procedure of risk assessment

4.4   Risk assessment methods

Several steps are required to perform a PRA: [1]

● Identify a spectrum of **initiating events**[A]– things that could possibly cause the hazard(such as the **loss of offsite power event**[A])

● Estimate the **frequency** of each initiating event by answering questions

● Specify the **hazard** – the outcome(s) to be prevented or reduced. For nuclear power plants, the focus is reducing the chance of damaging to the reactor core and potential release radioactive material to the environment or the public.

The risk analysts assume each possible initiating event occurs and then in response to that event, realistically identify each combination of failures (e.g., pump failure and valve failure), or "sequence," that leads to a specific outcome (e.g. core damage). This procedure is performed by the **Event Tree.**   Analysts then calculate the likelihood of all the sequences that lead to the same outcome. The likelihood of the outcome is the sum of the sequence frequencies. This procedure is implemented by the **Fault Tree**. The examples of event tree and fault tree will be made in **section 4.5**.

Besides the Event Tree and the Fault Tree, the technique of **Human reliability analysis** is also used to for PRA.

Human reliability analysis evaluates human errors that are important to the outcome of an event. Analysts assess the probability of human mistakes in light of factors such as training, procedures, and expected conditions during an event.

Figure 4-2 shows the whole procedure of risk assessment. [4]
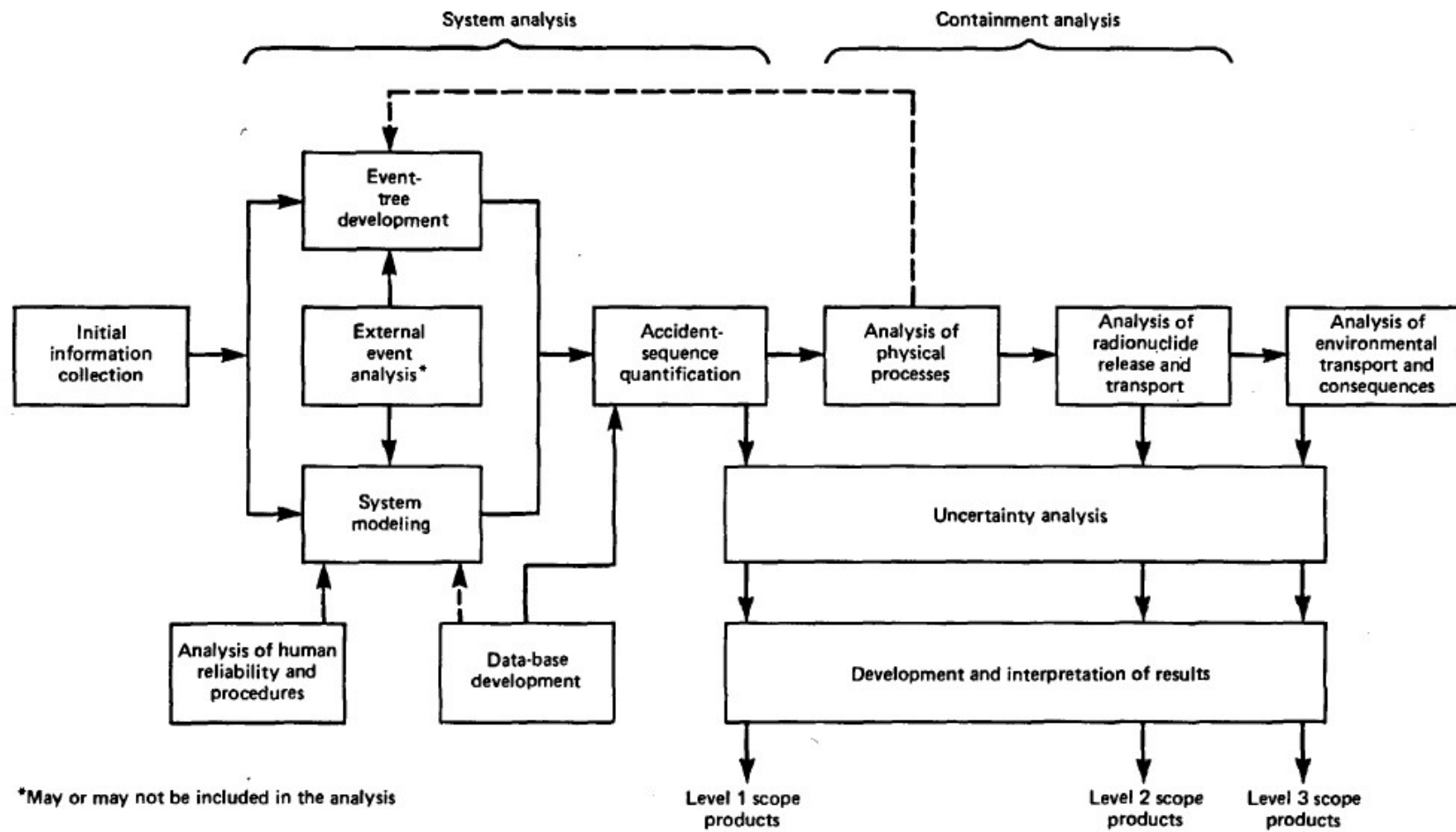
Figure 4-2 Procedure of risk assessment

4.5   Event tree and fault tree

4.5.1   Event tree

Event trees are used to model the sequence of events from an initiating event to an end state. The events in the event tree, which graphically represent the systems needed to keep the plant in a safe state after an initiating event occurring, are called **top trees**.

A graphical depiction of a sequence of events is shown in Figure 4-3 and an example of core damage sequence is shown in Figure 4-4 [5].

	Reactor trip	Hi Pressure Injection	Reduce Pressure	Lo Pressure Injection	
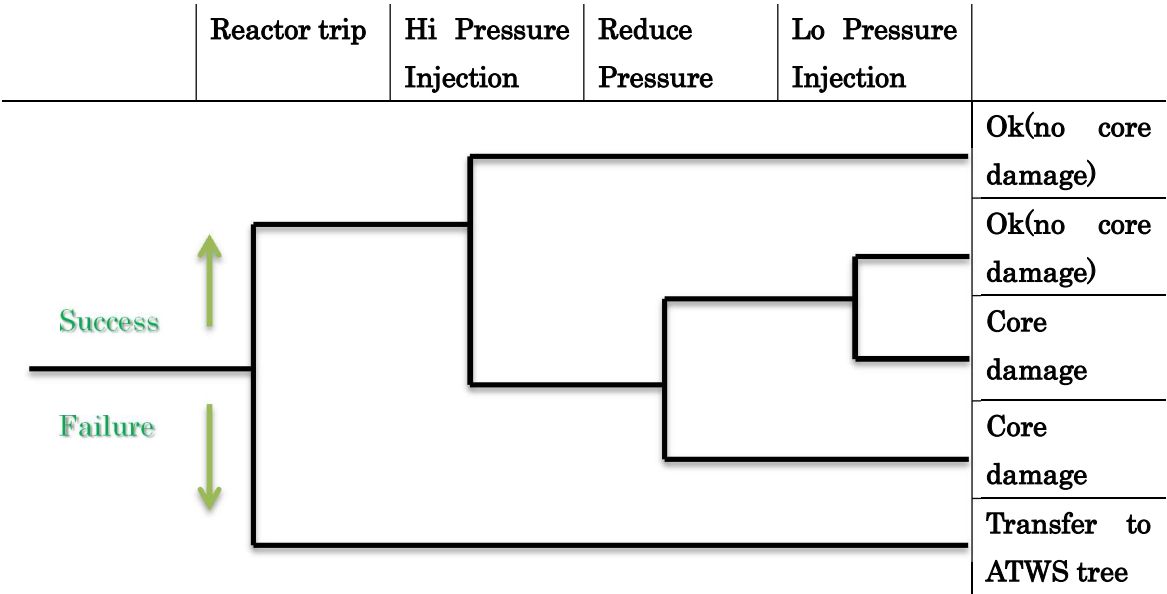
Figure 4-3   A depiction of Event Tree
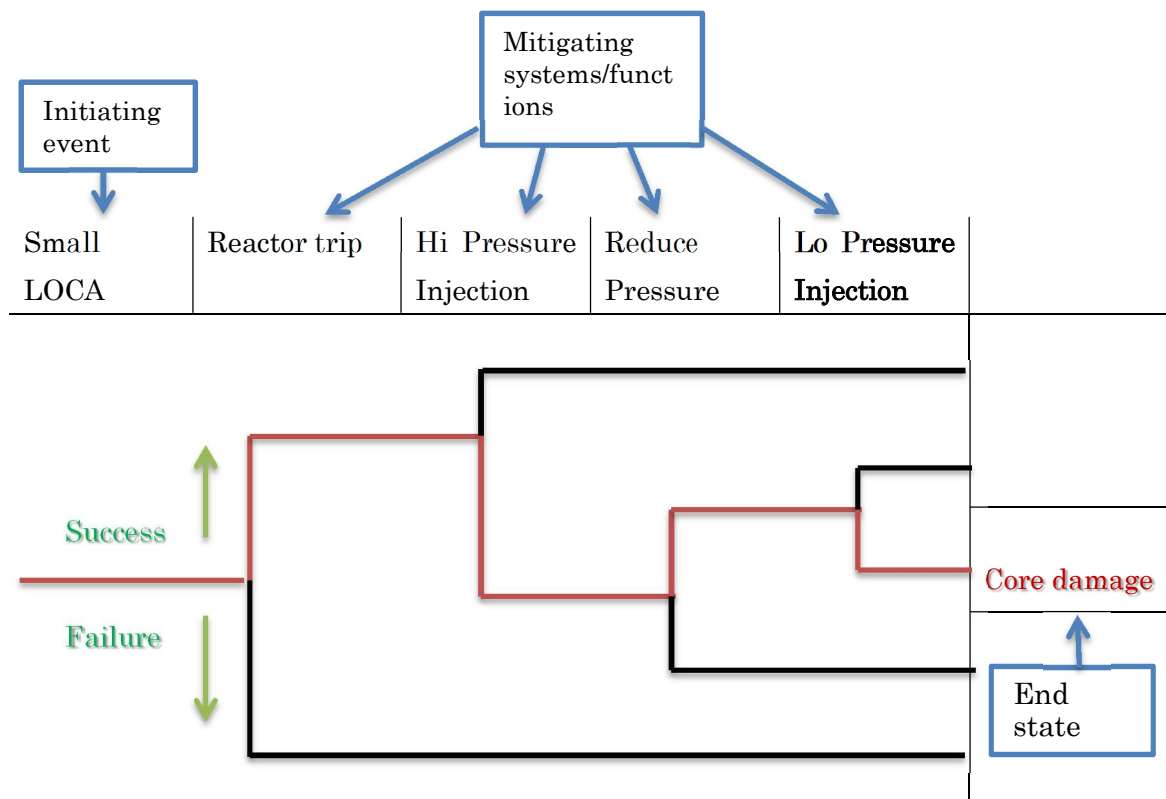
Figure 4-4　An example of core damage sequence

The core damage sequence:

Small LOCA occurs ➡ Reactor trip succeeds ➡ High pressure injection fails ➡ Reducing pressure succeeds ➡ Low pressure injection fails

4.5.2 Fault tree

A fault tree identifies all of the pathways that lead to a system failure. Toward that end, the fault tree starts with the top event, as defined by the event tree, and identifies (using the AND, OR, M out of N logic connectors) what equipment and operator actions, if failed, would prevent successful operation of the system.

Here we make a depiction of fault tree starting with the top event, i.e. low-pressure injection fails. **Figure 4-5** shows the success criterion of low-pressure injection fails. Moreover, Fault tree for low-pressure injection fails is shown in **Figure 4-6** [5]
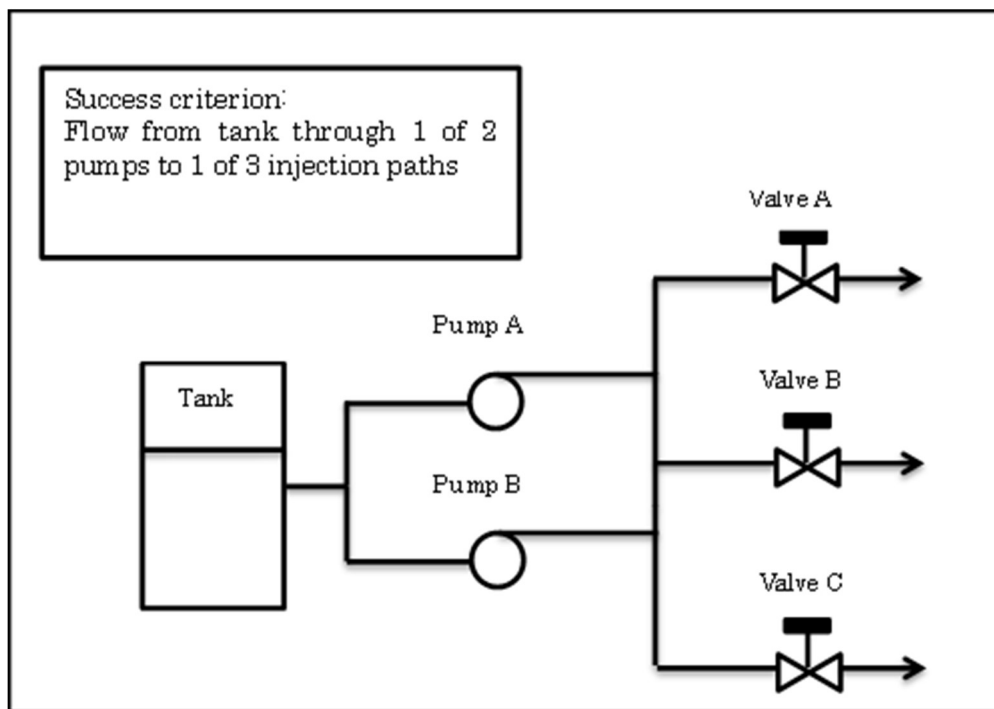
Figure 4-5   Success criterion
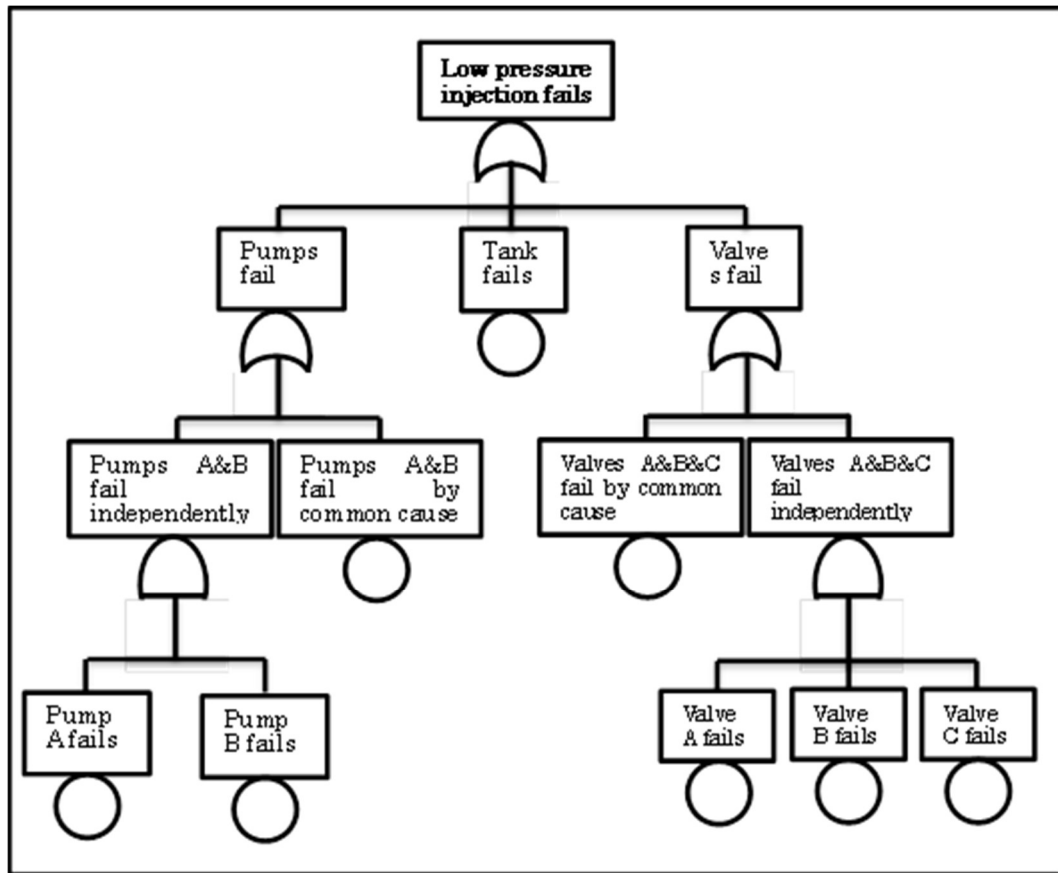
Figure 4-6   Fault tree for low pressure injection fails

Where,

Low-press
ure   →   Top event: system/function failure from event

Valves
A&B&C fail
by   common   →   **Common cause failure**[A]: one mechanism fails
                       all components in a group

Pump
B fails   →   Basic event: equipment or human failure for
                       which we have data

⌒   →   OR gate: a failure of any input causes overall

⌒   →   AND gate: all inputs must fail to cause overall
                       failure
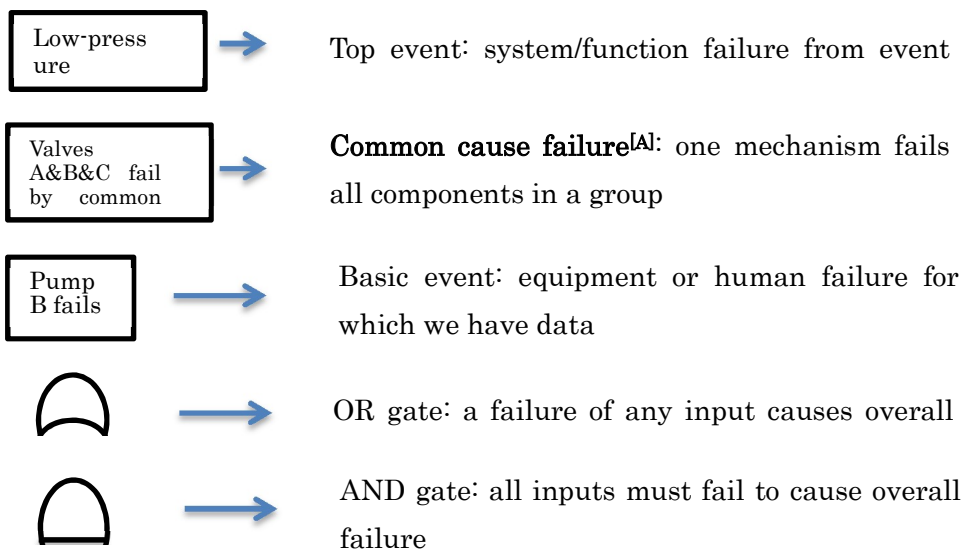
4.6 Fire PRA Methodology

4.6.1 Background

A number of fire PRA approaches were published during the 1990s (NUREG/CR-2300, 1983, NUREG/CR-2815, 1985, NUREG/CR-4840, 1990 and NUREG-1407, 1991 and so on). These approaches have generally the same structure. Their differences lie primarily in the underlying assumptions, analytical methods, tools, and data used. The overview of the common fire PRA structure is provided in **NFPA 805**.

Among those PRA approaches, the Individual Fire Examinations of External Event (IPEEE) program has used the fire PRA to facilitate a nuclear power plant examination for vulnerabilities **(NUREG-1407)**. Benefitting from the lessons learned from this program, in order to make finer, more realistic decisions for risk-informed regulation, Fire PRA methods needed to be improved. In order to address the need for improved methods, the Electric Power Research Institute (EPRI) and the U.S. Nuclear Regulatory Commission (NRC) office of Nuclear Regulatory Research (RES) initiated a collaborative project under the terms of an NRC/EPRI Memorandum of Understanding. The collaboration, known as the Fire Risk Quantification Study, has resulted in **state-of-the-art** [A] methods, tools, and data for a fire probabilistic risk assessment (PRA) for commercial nuclear power plant application that were documented in EPRI/NRC-RES Fire PRA Methodology for Nuclear Power Facilities, **NUREG/CR-6850.**

4.6.2 Fire PRA (PSA) process

According NFPA 805 [6], Fire PSA is a process to develop a plant's fire risk and fire safety insights based on the plant's design, layout, and operation. The process contains analysis elements that correspond directly to the elements of fire protection **defense-in-depth** [A], as follows:

(1) Fire initiation
(2) Fire growth (including detection, suppression, and confinement) and consequential equipment/**circuit damage**
(3) Post-fire safe shutdown

Note) the term of Probabilistic Safety Assessment (PSA) is also referred to as a probabilistic risk assessment (PRA).

A fire PRA requires a team effort because few individuals have the full range of expertise and knowledge necessary to complete the analysis. The needed areas of expertise are as follows: [7]

(1) *Fire analysis* (basic fire behavior, fire modeling, fire protection engineering, and plant fire protection regulatory compliance practices and documentation);
(2) *General PRA and plant systems analysis* (event tree/fault tree analysis, nuclear power plant systems modeling, reliability analysis, PRA practices as applied in the internal events domain, and specific knowledge of the plant under analysis);
(3) *Human reliability analysis* (emergency preparedness, plant operations, plant-specific safe shutdown procedures, and operations staff training practices);
(4) *Electrical analysis* (circuit failure modes and effects analysis and post-fire safe shutdown, including plant-specific regulatory compliance strategies and documentation).

The overview of the common fire PRA structure [6], as previously noted, is discussed in **NFPA 805** as follows:

A fire PSA is generally performed in stages. For the purpose of illustration, three stages of analysis are defined: qualitative screening, quantitative screening, and detailed analysis.

Qualitative screening

During qualitative screening, the plant is divided into fire areas and the potential impact of an unsuppressed fire on nuclear safety is considered. With substantiation, the qualitative screening analysis can also be refined to the consideration of fire zones rather than complete fire areas. The screening process includes consideration of potential multiarea or multizone fire effects. This stage of analysis is primarily dependent on the mapping of plant systems and components (including instrument, control, and power cables) to specific fire areas/zones. Qualitative screening considers the possibility that equipment losses due to fire in a given fire area/zone could lead to nuclear safety challenges. Nuclear safety challenges involve damage to nuclear safety targets or equipment that can potentially result in a plant transient. Fire areas and/or fire zones where a fire scenario cannot lead to nuclear safety challenges can be qualitatively screened and no further analysis is required for these areas/zones.

Quantitative screening

In the quantitative screening stage, fire areas and/or fire zones that survive qualitative screening are reconsidered using quantitative methods of limited depth and complexity. The quantitative screening stage limits consideration to two quantitative factors: namely, the overall frequency of fires and the conditional core damage probability (CCDP) assuming loss of all equipment in the impacted areas or zones. The product of these two factors provides the preliminary screening CDF for that area/zone. Quantitative screening criteria are established to ensure that an acceptable fraction of the total fire-induced CDF is captured. Fire areas and/or zones whose contributions to CDF fall below the established quantitative screening criteria are screened from further analysis.

At this stage of analysis, features or systems that require more extensive supporting engineering evaluations are generally not credited. Intervention by detection and suppression activities and other features or systems that might limit the extent of fire growth or damage are treated in the detailed analysis. These considerations are deferred to the detailed analysis.

Detailed analysis

For fire areas/zones that survive quantitative screening, further analysis is undertaken to more accurately and realistically quantify the fire area/zone risk contributions. The detailed analysis is also used as a supplemental screening tool. If at any time during

this stage of analysis the fire area/zone risk contribution is shown to be below the established quantitative screening criteria, then the analysis of that fire area/zone can be considered complete.

The detailed analysis is supported by engineering evaluation and fire modeling as appropriate, and any and all fire protection features and factors that could impact the postulated scenarios can be considered. These can include detection, suppression, fire source intensity, fire growth behavior, the timing and extent of fire damage, plant response, and operator actions that might mitigate the nuclear safety consequences of a fire.

In detailed quantification, a number of individual fire scenarios can be analyzed (where each scenario represents a postulated fire source in a specific plant location). Specific fire behaviors important to each postulated scenario are considered.   '

Fire PRA can vary in the number and definition of the stages employed. So the three stages above just address the general Fire PRA structure not serve as a fire PRA procedure guide. The **NUREG/CR-6850 (2005)** which documented state-of-the-art methods, tools, and data for the conduct of a fire PRA will provide a structured framework for conduct of the overall Fire PRA, as well as specific recommended practices to address each key aspect of the analysis.

Figure 4-7 and **Figure 4-8** show the overview of the Fire PRA process.[7] In the use of these figures something important should be noticed that a fire PRA is iterative, that is, certain tasks may need refinement after conduct of one or more of the subsequent task.
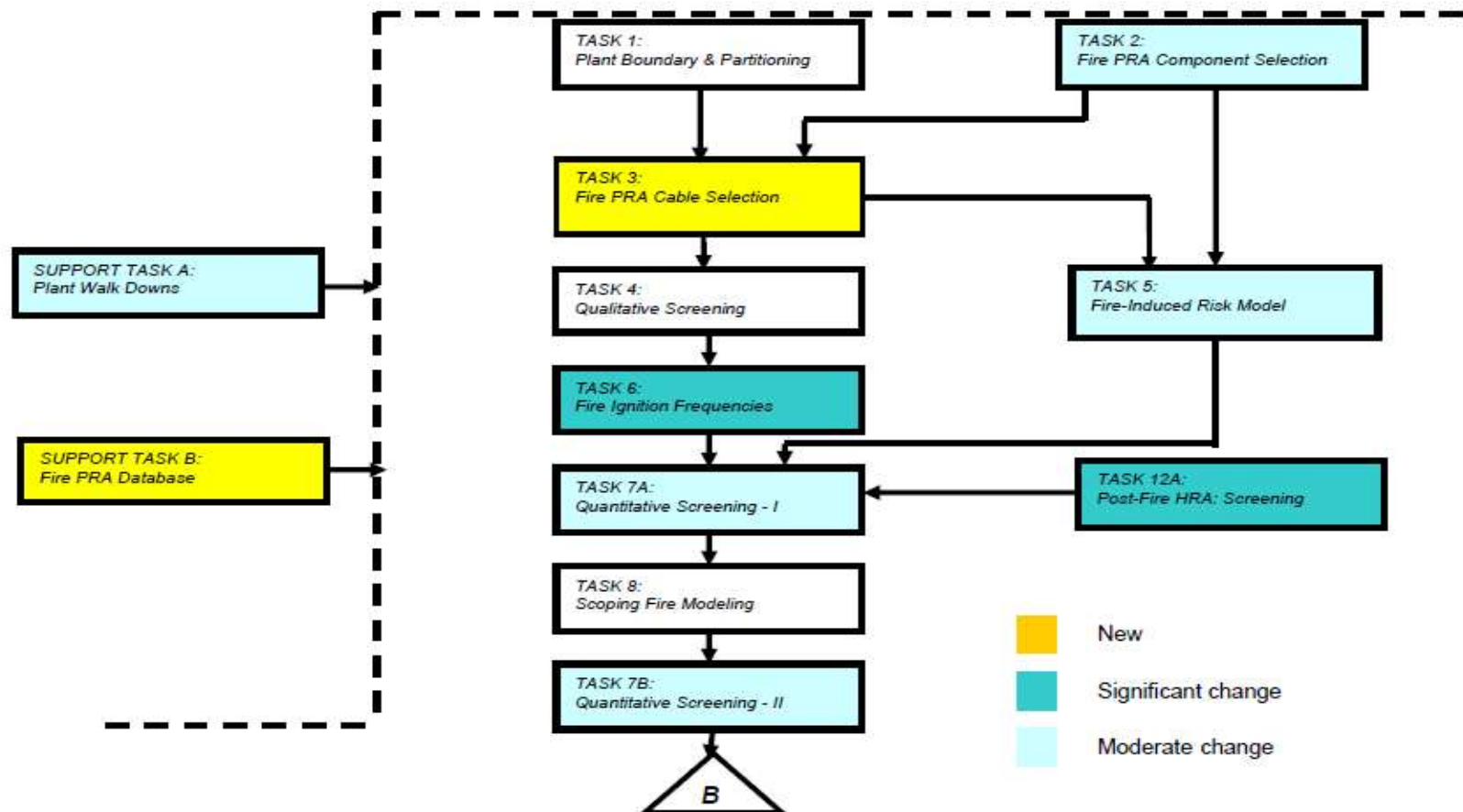
Figure 4-7 Overview of the Fire PRA Process
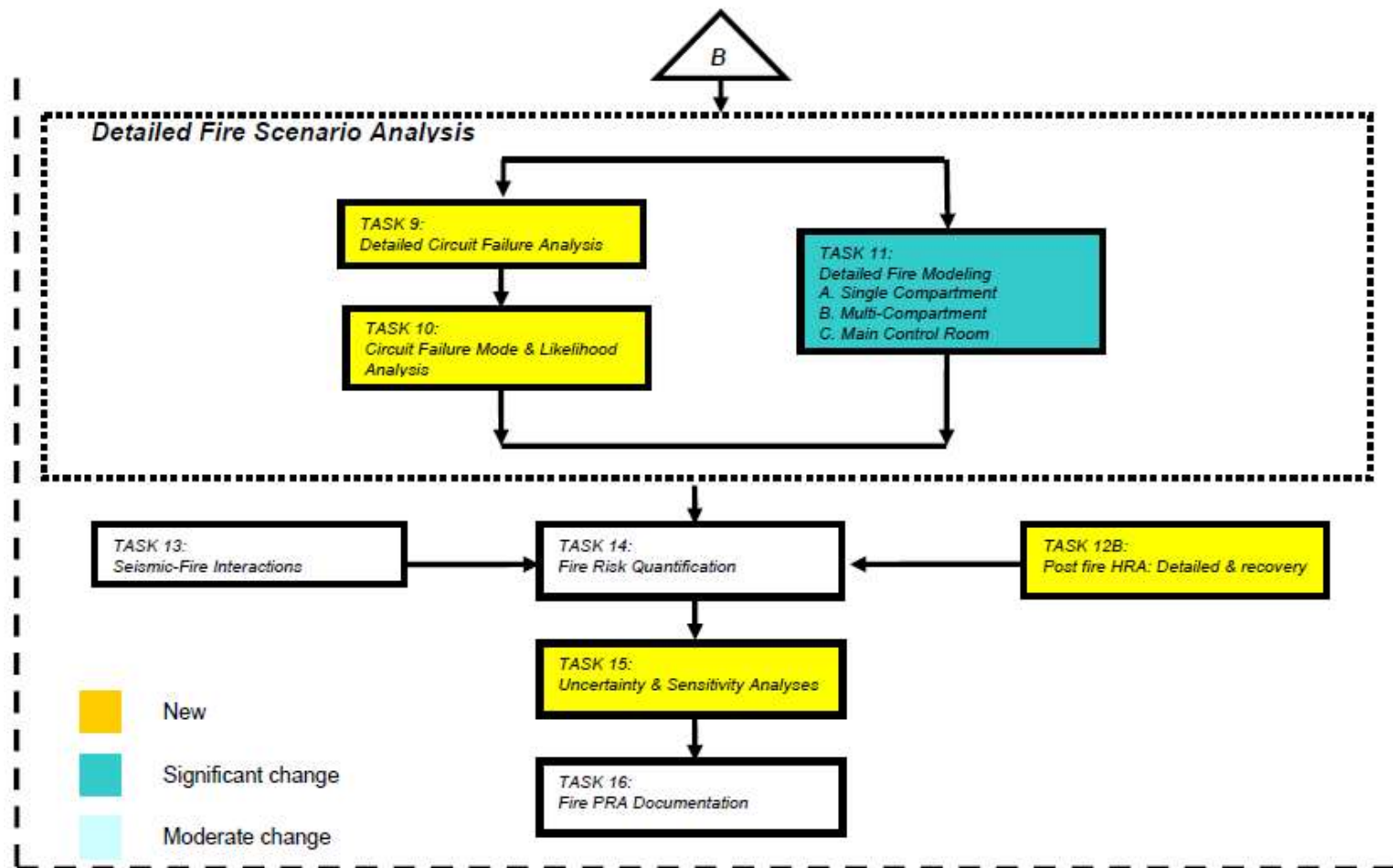
Figure 4-8   Overview of the Fire PRA Process (continued)

Among these tasks, the tasks in concert with the three steps of Fire PRA in NFPA 805 are Qualitative Screening (Task 4), Quantitative Screening (Task7), and Detailed Fire Scenario Analysis (Detailed Circuit Failure Analysis (Task 9), Circuit Failure Mode Likelihood Analysis (Task 10) and Detailed Fire Modeling (Task 11)) Some other tasks could be considered as the supplements to these tasks. [A] (* The descriptions of these tasks are discussed in Appendix)

In **NFPA 805** [6],
'A fire PSA is a process by which fire-induced contributions to plant risk are identified and quantified. During this process the plant is divided into **fire areas and/or fire zones.** In each fire area/zone, fire event scenarios are postulated and analyzed. In a direct quantification of fire risk, each fire area/zone is either screened from further consideration or quantified to estimate the fire risk. '

In **NUREG/CR-6850** [7],
*'For the purposes of a Fire Probabilistic Risk Assessment (PRA), the plant is divided into a number of **fire compartments**. The analysis then considers the impact of fires in a given compartment, and fires that might impact multiple compartments. This procedure establishes the process for defining the global plant analysis boundary and partitioning of the plant into fire compartments. The product of this task will be a list of plant fire compartments in the nuclear power plant under analysis. '*

We can find that the Fire PRA conducted in **NUREG/CR-6850** is based on the **fire compartments instead of** the **fire areas or fire zones** used in NFPA 805.

According to the **NUREG/CR-6850**[7], the Fire PRA always initially consider the Fire threats to safe shutdown in the context of the fire compartments which act as fundamental basis of the subsequent Fire PRA. **Fire compartment** is an area which is bounded by non-combustible barriers (not equal to fire barrier) where heat and products of combustion from a fire can be well confined within the enclosure. The term fire compartment is defined specifically for fire risk analysis and maps plant fire areas and/or zones into compartments defined by fire damage potential. Fire compartments generally fall within a fire area, meanwhile, the plant **Fire Hazards Analysis (FHA)** may also identify fire zones within the fire areas. Thus sometimes fire zone can be accepted as equivalent to fire compartments. However, care should be exercised in this because the fire zones may not satisfy the fire compartment definition.

The definitions of fire area, fire zone and fire compartment in NUREG/CR-6850 are listed in **Table 4-4** through **Table 4-6.** The important points of these concepts are listed in **Table 4-1 .**

Table 4-1　Important points of three concepts

NUREG/CR-6850		
➢ **Fire area**	**Fire zone**	**Fire compartment**
➢ Portion of a building or plant ➢ Separated from other areas by boundary fire barriers. ➢ Adequate for the fire hazard	➢ Subdivisions of fire areas ➢ Not necessary bounded by fire-rated assemblies	➢ A subdivision of a building or plant ➢ Not necessarily with fire barriers ➢ Bounded by non-combustible barriers where confine the heat and products of combustion
	➢ Often defined based on the fire suppression and/or detection systems ➢ Defined in the context of the fire protection program	➢ Specifically for fire PRA (for fire risk analysis) ➢ A well-defined enclosed room ➢ Generally fall within a fire area ➢ A fire compartment may contain one or more fire zones. ➢ Maps plant fire areas and/or zones into compartments

In addition, fire compartment partitioning situations are simplified shown in **Figure 4-9.**[7]

(1) Individual fire areas can be retained in total as fire compartments without further partitioning.

(2) However, with proper justification, a fire area may be partitioned into two or more fire compartments.

(3) In some rare cases, it may also be advantageous to combine two or more fire areas into a single fire compartment, particularly if the combined compartment is expected to have a minimal risk contribution (e.g., it may

screen at an early stage of the analysis).

(1)
Fire area
=
Fire compartment

(2)
Fire compartment | Fire compartment
Fire compartment
Fire area

Rare cases:

(3)
Fire area | Fire area
Fire area
Fire compartment
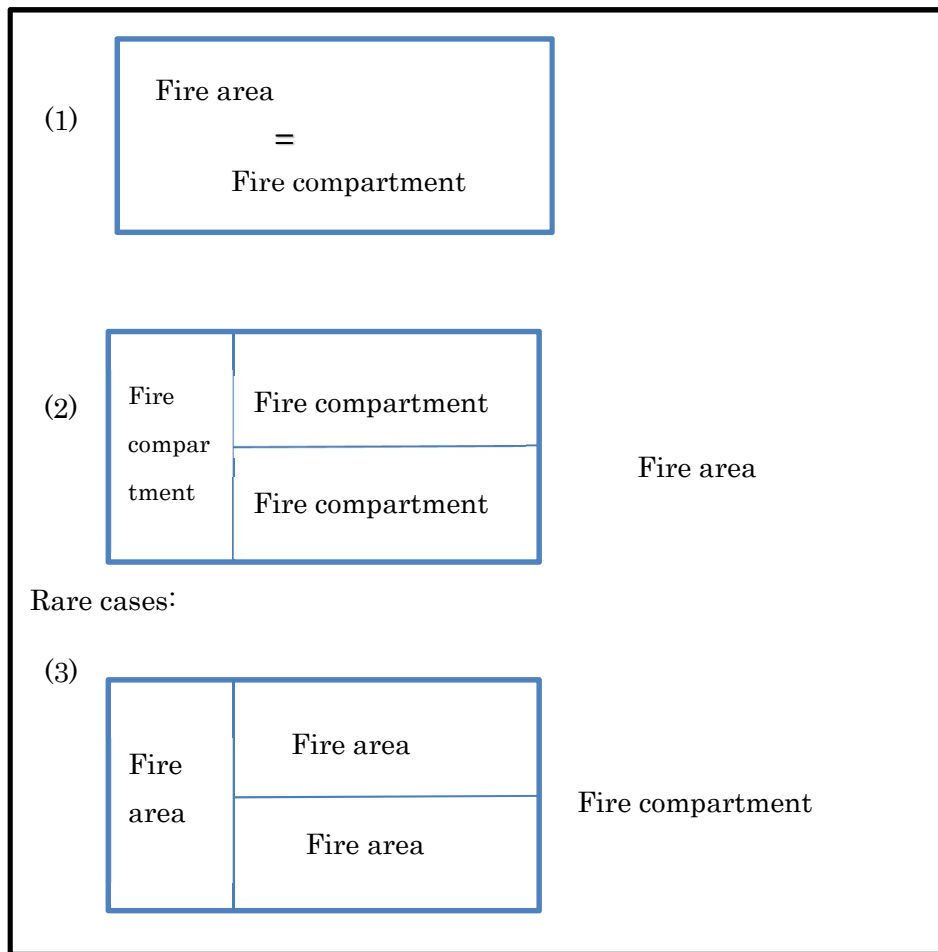
Figure 4-9　Fire compartment partitioning

4.7   Four terms in relevant documents

One of the most important effects of the plant partitioning process (Task 1) is related to the qualitative and quantitative screening tasks. Thus, the definition of fire compartments is significant to the analysis. However, after searching the relevant documents, I find that the term of **Fire compartment** is just used in NUREG/CR-6850 for Fire PRA. The definitions of related terms of fire area, fire zone, and fire barrier may have been improved with time going or have been revised for using in different situation. In this case, they may have subtle differences in these documents. The four terms and relevant documents are listed in Table 4-2. These documents are picked up from the documents referred previously which contain these four terms. The definitions of these four terms in respective document are listed in **Table 4-3** through **Table 4-6**.

As noted in Chapter 3, the BTP APCSB 9.5-1 and CMEB 9.5-1 are two guidelines for fire protection for Nuclear Power Plant (NPP) published in May 1976 and July 1981 respectively. The Regulatory Guide 1.189 provides guidance for Deterministic Fire Protection. NFPA 805 and NUREG/CR-6850 are used for NPP to apply risk-informed, performance-based requirements.

Table 4-2   Four terms used in fire protection and the relevant documents

Documents	Terms
1 BTP APCSB 9.5-1(1976.05~)	Fire barrier
2 CMEB 9.5-1(1981.07~)	Fire area
3 Regulatory guide 1.189(2001~)	Fire zone
4 NFPA 805(2001.01~)	Fire compartment
5 NUREG/CR-6850(2005.9~)	

4.7.1 Fire barrier

Table 4-3　Definitions of Fire Barrier

Documents	Definitions
BTP APCSB 9.5-1	Those components of construction (walls, floors, and roofs) that are rated by approving laboratories in hours for resistance to fire to prevent the spread of fire.
CMEB 9.5-1	Those components of construction (walls, floors, and their supports), including beams, joists, penetration seals or closures, fire doors and fire dampers that are rated by approving laboratories in hours of resistance to fire and are used to prevent the spread of fire.
Regulatory guide 1.189	Components of construction (walls, floors, and their supports), including beams, joists, columns, penetration seals or closures, fire doors, and fire dampers, that are used to prevent the spread of fire and that are rated by approving laboratories in hours of resistance to fire.
NFPA 805	A continuous vertical or horizontal construction assembly designed and constructed to limit the spread of heat and fire and to restrict the movement of smoke.
NUREG/CR-6850	Components of construction (walls, floors, and their supports), including beams, joists, columns, penetration seals or closures, fire doors, and fire dampers that are rated by approving laboratories in hours of resistance to fire, that are used to prevent the spread of fire (per U.S. NRC RG 1.189) and restrict spread of heat and smoke.

4.7.2　Fire area

Table 4-4　Definitions of Fire area

Documents	Definitions
BTP APCSB 9.5-1	That portion of a building or plant that is separated from other areas by boundary fire barriers (walls, floors or roofs) with any openings or penetrations protected with seals or closures having a fire resistance rating equal to that of the barrier.
CMEB 9.5-1	that portion of a building or plant that is separated from other areas by boundary fire barriers
Regulatory guide 1.189	The portion of a building or plant that is separated from other areas by rated fire barriers adequate for the fire hazard.
NFPA 805	An area that is physically separated from other areas by space, barriers, walls, or other means in order to contain fire within that area.
NUREG/CR-6850	The portion of a building or plant that is separated from other areas by rated fire barriers adequate for the fire hazard (per U.S. NRC regulatory guide 1.189).

4.7.3　Fire zone

Table 4-5　Definitions of Fire zone

Documents	Definitions
BTP APCSB 9.5-1	that portion of a building or plant that is separated from other areas by boundary fire barriers (walls, floors or roofs) with any openings or penetrations protected with seals or closures having a fire resistance rating equal to that of the barrier.
CMEB 9.5-1	that portion of a building or plant that is separated from other areas by boundary fire barriers
Regulatory guide 1.189	The portion of a building or plant that is separated from other areas by rated fire barriers adequate for the fire hazard.
NFPA 805	An area that is physically separated from other areas by space, barriers, walls, or other means in order to contain fire within that area.
NUREG/CR-6850	The portion of a building or plant that is separated from other areas by rated fire barriers adequate for the fire hazard (per U.S. NRC regulatory guide 1.189).

4.7.4 Fire compartment

Table 4-6　Definition of Fire compartment

Documents	Definitions
BTP APCSB 9.5-1	-
CMEB 9.5-1	-
Regulatory guide 1.189	-
NFPA 805	-
NUREG/CR-6850	A subdivision of a building or plant defined **specifically** for the purpose of fire PRA. A fire compartment is a well-defined enclosed room, **not necessarily** with fire barriers. They generally fall within a fire area, and are bounded by **non-combustible barriers** where heat and products of combustion from a fire within the enclosure will be substantially confined. Boundaries of a fire compartment may have open equipment hatches, stairways, doorways or unsealed penetrations. This is a term defined specifically for fire risk analysis and maps plant fire areas and/or zones, defined by the plant and based on fire protection systems design and/or operations considerations, into compartments defined by fire damage potential. For example, the control room or certain areas within the turbine building may be defined as a fire compartment.

4.8　Comparison of each term in different documents

Table 4-7 through **Table 4-9** shows the same points and different points of each term in different documents.

Table 4-7　Comparison of Fire barrier

	1 BTP APCSB 9.5-1 (1976.5~)	2 CMEB 9.5-1 (1981.7~)	3 Regulatory guide 1.189(2001~)	4 NFPA 805(2001~)	5 NUREG/CR-6850(2005~)
Fire barrier	➢ Components of construction ➢ Rated in hours for resistance to fire ➢ Prevent the spread of fire			➢ A continuous membrane either vertical or horizontal ➢ With a specified fire resistance rating ➢ Limit the spread of heat and fire ➢ Restrict the movement of smoke ➢ Could have protected openings	➢ (same as RG 1.189)
	➢ Walls, floors and roofs	➢ Walls, floors, and their supports			➢ (same as RG 1.189)
		➢ Including beams, joists, penetration seals or closures, fire doors and fire dampers			➢ (same as RG 1.189) ➢ Restrict spread of heat and smoke
			➢ Fire-resistance rating ≥3hours		➢ Fire protection endurance rating ≥ one hour
			➢ Exception: 1. Fire area boundaries need not be completely sealed floor-to-ceiling, wall-to-wall boundaries. 2. In this case, licensees should evaluate the adequacy of fire boundaries in their plants		

Table 4-8　Comparison of Fire area

	1　BTP　APCSB　9.5-1 (1976.5~)	2　　　CMEB 9.5-1 (1981.7~)	3　Regulatory　guide 1.189(2001~)	4 NFPA 805(2001~)	5　NUREG/CR-6850 (2005~)
Fire area	➢　　　Portion of a building or plant ➢　　　Separated from other areas by boundary fire barriers			➢　　　Physically　separated from　other　areas　by　space, barriers, walls, or other means	➢　　　(same as RG 1.189)
	➢　　　Fire　　barriers with　any　openings　or penetrations　　protected with　seals　or　closures having　a　fire　resistance rating equal to that of the barrier		➢　　　Adequate for the fire hazard	➢　　　Withstand　the　fire hazards　associated　with　the area ➢　　　Protect　　important equipment　within　the　area from a fire outside the area	

Table 4-9 Comparison of Fire zone

	1 BTP APCSB 9.5-1 (1976.5~)	2 CMEB 9.5-1 (1981.7~)	3 Regulatory guide 1.189(2001~)	4 NFPA 805(2001~)	5 NUREG/CR-6850(2005~)
Fire zone	➢ Subdivisions of fire areas				
	➢ In the suppression systems designed to combat particular types of fires	➢ Typically based on fire hazards analyses		➢ Not necessary bounded by fire-rated assemblies	
	➢ Fire zone concept helps the fire-fighter define the necessary fire parameters and actions			➢ Also refer to the area subdivisions of a fire detection or suppression system	➢ Often defined based on the fire suppression and/or detection systems ➢ Defined in the context of the fire protection program ➢ May contain one or more rooms

4.8.1 Discussion

About these five documents, we can divide them into two types based on their application: 1) As noted in Chapter 3, the **BTP APCSB 9.5-1, BTP CMEB 9.5-1 and Regulatory guide 1.189** are guidelines for fire protection for nuclear power plant to satisfy the traditional deterministic fire protection requirements. 2) The NFPA 805 and NUREG/CR-6850 are used for the performance-based fire protection. The performance-based fire protection approach is a new approach developed based on the traditional deterministic fire protection approach.

For fire barrier (Table 4-7), we can find that when the guideline developed to **Regulatory guide 1.189**, it uses "their supports" instead of "roofs", and also add the "beams, joists, penetration seals or closures, fire doors and fire dampers" to its definition, which make the definition be more detailed and clear. For deterministic fire protection, the fire-resistance rating of fire barrier needs to be equal to or larger than 3 hours. However, for performance-based fire protection, the minimum fire-resistance rating of fire barrier is reduced to one hour but the fire barrier is required to restrict spread of heat and smoke.

As referred previously, the Fire PRA conducted in **NUREG/CR-6850** is based on the **fire compartments instead of** the **fire areas or fire zones** used in NFPA 805. And care should be exercised in accepting fire zones as equivalent to fire compartments. Therefore, for the fire area (Table 4-8), the requirements of fire area for performance-based fire protection are almost same as that for deterministic fire protection. Combining **Table 4-1** and **Table 4-9**, we can find that the requirements of fire zone are more close to the requirements of fire compartment. For example, one of its both fire zone and fire compartment is not necessary bounded by fire-rated assemblies.

In a word, with the guidelines for fire protection improved, as well as the advanced fire protection approach developed, the definitions of fire area, fire zone, fire barrier are revised to satisfied different fire protection requirements.

Thereinto, the fire barriers which are the components of construction are rated in hours of resistance to fire and be used to prevent the spread of fire and even restrict spread of heat and smoke. They play significant role in the initial stage of fire protection. What's more, the assessment of the barrier failure probability is also one part of completing the quantitative analysis in PRA.

According to the **NUREG/CR-4840**[12], in the unscreened cut sets where a potential for barrier failure has been identified, barrier failure probability will be estimated using the barrier failure rates.

Barriers are grouped into three types: (1) fire doors, security doors, water-tight doors,

and fire curtains, (2) fire dampers and ventilation dampers; and (3) penetration seals and fire walls. The data base contains 628 records from when any plant began construction to the end of June 1985. The number of barriers of each type at a plant is not known precisely for each plant, but a nominal figure that has been estimated for each barrier type is given in **Table 4-10** . The uncertainty of each estimate is represented by 90% confidence bounds. These failure rate estimates and confidence bounds are given in **Table 4-11.**

Table 4-10   Approximate Number of Barriers at a plant

No.	Type	Nominal
1	Fire door, security doors, water-tight doors, and fire curtains	150
2	Fire dampers and ventilation dampers	200
3	Penetration seals and fire walls	3000

Table 4-11   Estimates of Single Barrier Failure Rate

Barrier type	Barrier/unit	Estimate	5% confidence bound	90% confidence bound
1	150	7.5E-3	0.0	2.4E-1
2	200	2.7E-3	0.0	2.2E-1
3	3000	1.2E-3	0.0	3.7E-2

The method for estimating the failure rates and quantifying the uncertainties in the estimates will be discussed in Chapter 5.

4.9　Supplement

● Compartmentalization and compartmentation in fire protection field

Compartmentalization can be described as a 'divide and conquer' process for separating thoughts that will conflict with one another. This may happen when they are different beliefs or even when there are conflicting values.[13] Besides, Compartmentalization or compartmentalisation may refer to compartmentalization in biology, engineering, fire protection, information security and psychology field. [3]

Compartmentation is defined as the division of a cell into different regions, either structurally or biochemically, based on dictionary. [14]

Compartmentalization or compartmentation [15][16] in structures is the fundamental basis and aim of passive fire protection.
The idea is to divide a structure into "fire compartments", which may contain single or multiple rooms, for the purpose of limiting the spread of fire, smoke and flue gases, in order to enable the three goals of fire protection:
- life safety
- property protection
- continuity of operations

What's more, in NUREG/CR-1.189 (2001) the description about the "compartmentation" is as follows:

'4.1.2 Compartmentation, Fire Areas and Zones

In accordance with GDC 3, structures, systems, and components important to safety must be designed and located to minimize the probability and effect of fires and explosions. The concept of compartmentation meets GDC 3, in part, by utilizing passive fire barriers to subdivide the plant into separate areas or zones. These fire areas or zones serve the primary purpose of confining the effects of fires to a single compartment or area, thereby minimizing the potential for adverse effects from fires on redundant structures, systems, and components important to safety. '

However, the revised Regulation guide 1.189 published in April 2009 used the "compartmentalization" instead of "compartmentation" without description changes [18] shown as follows:

'4.1.2 Compartmentalization, Fire Areas, and Zones

In accordance with GDC 3 (Ref. 1), SSCs important to safety must be designed and

located to minimize the probability and effect of fires and explosions. The concept of compartmentalization meets GDC 3, in part, by using passive fire barriers to subdivide the plant into separate areas or zones. The purpose of these fire areas or zones is to confine the effects of fires to a single compartment or area, thereby minimizing the potential for adverse effects from fires on redundant SSCs important to safety.'

Thus, in conclusion, compartmentalization could be considered as same as fire compartmentation in the fire protection field.

> ## Compartmentation of buildings [17]

Building Regulations "Approved document J, Combustion appliances and fuel storage systems", defines a **fire compartment** as:

'*… a building or part of a building comprising one or more rooms, spaces or storeys constructed to prevent the spread of fire to or from another part of the same building or an adjoining building. (A roof-space above the top storey of a fire compartment is included in that fire compartment.) A separated part of a building is a form of compartmentation in which part of a building is separated from another part of the same building by a compartment wall. Such walls run the full height of the part and are in one vertical plane.'*

Compartment walls and compartment floors form a complete barrier between fire compartments and are required to provide a minimum degree of fire resistance which is generally expressed in terms of the number of minutes of resistance that must be provided by different parts of a building. Doors within compartment walls, and other openings should have a similar fire resistance to the compartment walls or floors they penetrate.

Joints between fire-separating elements such as compartment walls or floors, should be fire-stopped to maintain the continuity of resistance; and openings for timber beams, joists, purlins and rafters, and pipes, ducts, conduits or cables that pass through any part of a fire-separating element should be kept as few in number as possible, as small as practicable; and should be fire-stopped.

NFPA 101 (Life safety code) which is the most widely used source for strategies to protect people based on building construction, protection, and occupancy features that minimize the effects of fire and related hazards define the Fire compartment as:

Fire compartment is a space within a building that is **enclosed by fire barriers** on all sides, including the top and bottom.

➢ **Fire compartmentation of nuclear power plant [7]**

Fire compartment is a subdivision of a building or plant defined **specifically** for the purpose of fire PRA.

A fire compartment is a well-defined enclosed room, **not necessarily with fire barriers**, which is different from the requirement in general building according to **NFPA 101**. Fire compartments generally fall within a fire area, and are bounded by non-combustible barriers where heat and products of combustion from a fire within the enclosure will be substantially confined. Boundaries of a fire compartment may have open equipment hatches, stairways, doorways or unsealed penetrations. The term fire compartment is defined specifically for fire risk analysis and maps plant fire areas and/or zones, defined by the plant and based on fire protection systems design and/or operations considerations, into compartments defined by fire damage potential. For example, the control room complex or certain areas within the turbine building may be defined as a compartment.

4.10 Appendix

A. Glossary

Common cause failure (CCF) [11]: A dependent failure in which two or more component fault states exist simultaneously, or within, a short time interval, and are a direct result of a shared cause.

Core damage frequency (CDF) [9]: An expression of the likelihood that, given the way a reactor is designed and operated, an accident could cause the fuel in the reactor to be damaged.

Defense-in-depth [6]: Protecting the safety of the public, the environment, and plant personnel from a plant fire and its potential effect on safe reactor operations is paramount to this standard. The fire protection standard shall be based on the concept of defense-in-depth. Defense-in-depth shall be achieved when an adequate balance of each of the following elements is provided:

Initiating event [8]: An initiating event is an unplanned event that occurs while a nuclear power plant is in critical operation and requires that plant to shut down to achieve a stable state.

Loss of offsite power (LOOP, also referred to as LOSP) event [10]: the simultaneous loss of electrical power to all unit safety buses (also referred to as emergency buses, Class 1E buses, and vital buses) requiring all emergency power generators to start and supply power to the safety buses. The nonessential buses may also be de-energized as a result of this.

(1) Preventing fires from starting
(2) Providing an adequate level of fire protection for structures, systems, and components important to safety, so that a fire that is not promptly extinguished will not prevent essential plant safety functions from being performed.
(3) Rapidly detecting fires and controlling and extinguishing promptly those fires that do occur, thereby limiting fire damage

State of the art [19]: refers to the highest level of general development, as of a device, technique, or scientific field achieved at a particular time. It also refers to such a level of development reached at any particular time as a result of the common methodologies employed at the time.

B. Fire Protection Documents

WASH-1400, NUREG/75-014: "Reactor Safety Study: An Assessment of Accident Risks in U.S. Commercial Nuclear Power Plants [NUREG-75/014 (WASH-1400)]", October 1975

NUREG/CR-2300: "PRA Procedures Guide-A Guide to the Performance of

Probabilistic Risk Assessments for Nuclear Power Plants", January 1983

NUREG/CR-2815: "Probabilistic Safety Analysis Procedures Guide", January 1984

NUREG/CR-4840: "Procedures for the External Event Core Damage Frequency Analyses for NUREG-1150", November 1990

NUREG-1407: "Procedural and Submittal Guidance for the Individual Plant Examination of External Events (IPEEE) for Severe Accident Vulnerabilities", June 1991

NFPA 805: "Performance-based Standard for Fire Protection for Light Water Reactor Electric Generating Plants", February 9, 2001

NFPA 101: LIFE SAFETY CODE (2015 edition)

NUREG/CR-6850: "EPRI/NRC-RES Fire PRA Methodology for Nuclear Power Facilities", September 2005

BTP APCSB 9.5-1: Branch Technical Position APCSB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants", May 1976

CMEB 9.5-1: Branch Technical Position CMEB 9.5-1, "Guidelines for Fire Protection for Nuclear Power Plants" (formerly Branch Technical Position ASB 9.5-1), July 1981.

Regulatory guide 1.189(2001~): "Fire Protection for Operating Nuclear Power Plants", April 2001

C. List of acronyms

NRC: U.S. Nuclear Regulatory Commission

EPRI: Electric Power Research Institute

NFPA: National Fire Protection Association

RES: office of Nuclear Regulatory Research (RES)

PRA (PSA): Probabilistic Risk Assessment (Probabilistic Safety Assessment)

IPEEE: Individual Fire Examinations of External Event (IPEEE)

CCDP: conditional core damage probability (CCDP)

CDF: core damage frequency

FHA: Fire Hazards Analysis

BTP: Branch Technical Position

INL: Idaho National Laboratory (The INL is a U.S. Department of Energy National Laboratory operated by Battelle Energy Alliance)

D. Some tasks of the Fire PRA (NUREG/CR-6850)

Task 4 Qualitative Screening

This procedure describes the criteria for qualitatively screening the fire compartments defined in Task 1.

From Task 1, Plant Partitioning, a set of fire compartments is identified for the Fire PRA. These compartments are subjected to a series of screening analyses that will determine the relative fire risk associated to each. Qualitative screening is the first of such screening analyses.

The results of this task, unscreened fire compartments, are used in:
· Task 6: Fire Ignition Frequency, where fire frequencies are estimated for each of the unscreened fire compartments; and
· Task 7: Quantitative Screening. The unscreened fire compartments are subjected to quantitative screening.

Task 7 Quantitative Screening

This section describes the procedure for performing the following quantitative screening tasks:

- · Task 7A–Quantitative Screening I

- · Task 7B–Quantitative Screening II

- · Task 7C–Quantitative Screening III (Optional)

- · Task 7D–Quantitative Screening IV (Optional)

This procedure provides the user an approach to quantify the Fire PRA Model using the procedure provided in Task 5 (Fire-induced risk model), and to screen out fire compartments based on quantitative criteria.

This procedure develops the bases for the quantitative screening criteria and provides specific methods for implementing the screening process.

Quantitative screening is primarily focused on a fire compartment level (i.e., Tasks 7A and 7B). Quantitative screening on a fire scenario level (i.e., Tasks 7C and 7D) is presented as optional tasks in this procedure.

Unscreened fire compartments from Task 7A are input to Task 8, Scoping Fire Modeling. Unscreened fire compartments from Task 7B are used in performing Task 11, Detailed Fire Modeling and Task 12, Post-Fire HRA, the detailed analysis portion. Optional Tasks 7C and 7D are performed in parallel with detailed fire

scenario analysis, and unscreened fire scenarios are input to Task 14, Fire Risk Quantification.

Task 9 Detailed Circuit Failure Analysis

Conducting a Fire PRA in accordance with this methodology necessitates an analysis of fire-induced circuit failures beyond that typically conducted during original Fire PRAs. The circuit analysis elements of the project are conducted in three distinct phases:

(1) Fire PRA **cable** selection (Task 3),

(2) Detailed **circuit** failure analysis (Task 9), and

(3) **Circuit** failure mode likelihood analysis (Task 10).

The purpose of Task 9 is to conduct a more detailed analysis of circuit operation and functionality to determine equipment responses to specific cable failure modes. These relationships are then used to further refine the original cable selection by screening out cables that cannot prevent a component from completing its credited function.

This task has inputs from Task 2 (Fire PRA Components Selection), Task 3 (Fire PRA Cable Selection), Task 7 (quantitative), Task11 (Detailed Fire Modeling) and Support Task B (Fire PRA Database System), and serve as input into the task 10 (Circuit Failure Mode Likelihood Analysis)

* The term "circuit" and "cable" are often used interchangeably for fire-related circuit analyses. A circuit is comprised of electrical components, subcomponents, and cables/connection wire. Within the context of fire-induced equipment failures, it is understood that "circuit failure" or "circuit response" refers to the impact of "cable failure modes" that may affect the behavior of related components and subcomponents in a complete circuit.

Task 10 Circuit Failure Mode Likelihood Analysis

This task conducts the third phase of circuit analysis stated in Task 9. Task 10 estimates the probability of hot short cable failure modes of interest, which in turn can be correlated to specific component failure modes.

This task needs inputs from Task 3 (Fire PRA Cable Selection), Task 9 (Detailed Circuit Failure Analysis), Tasks 11 and 14 (Detailed Fire Modeling and Quantification of Fire Risk), and Support Task B (Fire PRA Database System). The circuit failure probability estimates also serve as inputs to the detailed fire scenario quantification process (Task 11). The results of this task might also be used in Task 12 (Post-fire HRA).

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4.11 References

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5 Reliability Analysis of the building equipment data utilizing the methodology of parameter estimation for PRA in nuclear power plant [A]

PRA is a mature technology that can provide a quantitative assessment of the risk from accidents in nuclear power plants. The data analysis portion of a nuclear power plant PRA provides estimates of the parameters used to determine the frequencies and probabilities of the various events modeled in a PRA.[2]   In addition, so far as I know, there is a **periodical inspection reporting system** [A] for inspecting operation statement of the fire prevention equipment. However, it seems like that no quantitative analysis is carried out for these data from the inspection.

Thus, after studying the knowledge of PRA and the parameter estimation for PRA, I consider that if the methodology applied in nuclear power plant could be utilized in the common buildings.

As stated in Chapter 4, this chapter will focus on the failure rate. However, since the data of fire barrier in buildings is limited, the data of the emergency lighting equipment which also very a significant role in fire prevention will be used instead. Therefore, based on the knowledge of PRA in nuclear power plant, this chapter will estimate the failure rates of the emergency lighting equipment and quantify the uncertainties in the estimates utilizing the methodology of parameter estimation for PRA.

5.1   Data introduction

The objects of this analysis are the emergency lighting systems, installed in the buildings in X prefecture, Tokyo, which were indicated to be corrected in 2014, which, however, were without indication (no need correction) in 2013 based on the **Periodic inspection and report system**. Here we have 107 buildings like that in total. [4]

We pick the number of the emergency lighting equipment that was inspected, X_i and the number of broken emergency lighting equipment, x_i during one year in each building.

In this research, as the data are limited, I will regard the number of emergency lighting equipment as the time period of the inspection of emergency lighting in each building, in another word, these data mean that there are x_i failed emergency lighting equipment in t_i $(X_i$) years. And the corresponding λ_i, equaling to x_i/t_i, is the rough failure rate of the emergency lighting equipment in each building. The unit of the failure rate λ_i is failures per year.

The meanings and the values of these parameters are recorded in **Table 5-1 and Table 5-2** respectively. The data recorded in Table 5-2 are ordered by failure rate λ_i

ascending.

Note) [5]:

➢ The measurement results of the illumination of the emergency lighting equipment is record in Appended Table 4(別表 4) of the periodic inspection report. The flow path from the lighting inspection of the emergency light equipment to the record way of the Appended Table 4 is as follows,

　　1. Perform lighting inspections on all luminaire. Then focus on the parts that are important for refuge (such as corridor, staircases, emergency elevator hall and the entrance of the living room and so on) and carry out the illumination measurement.

　　2. Extract the minimum illumination of each type of light source in each floor from the results of illumination measurement in plural places, and record the measurement location, illumination and so on in attached sheet(別紙).

➢ About the inspection of the emergency lighting equipment, there is something should pay attention to:
2(1) the change to auxiliary power and the situation of the lighting of the luminaire devices:
Confirm that the power supply is about to change to the auxiliary power immediately and automatically and then the emergency lights turn on when the commercial power supply is cut off. What's more, confirm that the auxiliary power can also be restored automatically in the case that the commercial power supply is restored.

Note) the lighting inspection is carried up on all the luminaire installed in the target buildings.

Table 5-1　Meanings of parameters

No.	Inspected building number in X prefecture in Tokyo city
t_i	Time period of the inspection of emergency lighting in each building (Total number of inspected emergency lighting equipment in each building)
x_i	Number of lights that needed correction
$\lambda_i = x_i / t_i$	Rough failure rate of the emergency lighting equipment

Table 5-2　Values of parameters

No.	1	2	3	4	5	6	7	8	9	10
t_i	31	27	80	25	150	24	21	20	20	19
x_i	1	1	3	1	6	1	1	1	1	1
λ_i	0.032	0.037	0.038	0.040	0.040	0.042	0.048	0.050	0.050	0.053

No.	11	12	13	14	15	16	17	18	19	20
t_i	38	32	16	15	30	15	15	14	14	27
x_i	2	2	1	1	2	1	1	1	1	2
λ_i	0.053	0.063	0.063	0.067	0.067	0.067	0.067	0.071	0.071	0.074

No.	21	22	23	24	25	26	27	28	29	30
t_i	13	13	13	38	12	12	12	12	155	23
x_i	1	1	1	3	1	1	1	1	13	2
λ_i	0.077	0.077	0.077	0.079	0.083	0.083	0.083	0.083	0.084	0.087

No.	31	32	33	34	35	36	37	38	39	40
t_i	22	11	11	10	10	10	29	38	19	9
x_i	2	1	1	1	1	1	3	4	2	1
λ_i	0.091	0.091	0.091	0.100	0.100	0.100	0.103	0.105	0.105	0.111

No.	41	42	43	44	45	46	47	48	49	50
t_i	9	9	9	9	9	44	8	16	23	23
x_i	1	1	1	1	1	5	1	2	3	3
λ_i	0.111	0.111	0.111	0.111	0.111	0.114	0.125	0.125	0.130	0.130

No.	51	52	53	54	55	56	57	58	59	60
t_i	15	15	15	7	14	14	7	28	7	13
x_i	2	2	2	1	2	2	1	4	1	2
λ_i	0.133	0.133	0.133	0.143	0.143	0.143	0.143	0.143	0.143	0.154

No.	61	62	63	64	65	66	67	68	69	70
t_i	13	18	17	17	17	17	22	11	37	5
x_i	2	3	3	3	3	3	4	2	7	1
λ_i	0.154	0.167	0.176	0.176	0.176	0.176	0.182	0.182	0.189	0.200

No.	71	72	73	74	75	76	77	78	79	80
t_i	10	9	13	82	8	8	8	8	20	8
x_i	2	2	3	19	2	2	2	2	5	2
λ_i	0.200	0.222	0.231	0.232	0.250	0.250	0.250	0.250	0.250	0.250

No.	81	82	83	84	85	86	87	88	89	90
t_i	12	19	15	15	11	11	11	10	10	19
x_i	3	5	4	4	3	3	3	3	3	6
λ_i	0.250	0.263	0.267	0.267	0.273	0.273	0.273	0.300	0.300	0.316

No.	91	92	93	94	95	96	97	98	99	100
t_i	6	9	9	18	3	14	11	10	15	14
x_i	2	3	3	6	1	5	4	4	6	6
λ_i	0.333	0.333	0.333	0.333	0.333	0.357	0.364	0.400	0.400	0.429

No.	101	102	103	104	105	106	107			
t_i	9	4	2	10	17	3	13			
x_i	4	2	1	5	11	2	9			
λ_i	0.444	0.500	0.500	0.500	0.647	0.667	0.692			

5.2 Initiating events model

There are several **probability models** [A] that used for probabilistic risk assessment (PRA) in nuclear power plant. One of them is the initiating events model.

Initiating event [A] is an event, either internal or external to the plant, which triggers a sequence of events that challenge plant control and safety systems, whose failure could potentially lead to core damage or large early release.

Table 5-3 indicates the initiating events model, and its parameters and the data needed for analysis.

* Initiating events occur infrequently in nuclear power plants. Of the 16 initiating event categories currently trended, only five have prediction limit event counts of 6 or more. The initiating events in this category are labeled "infrequent." The other initiators (which have event count of 2 or 3), are labeled "**sparse**." [1]

Table 5-3   Initiating events model [2]

Typical event	Event occurs initiating an accident sequence
Parameter(s) to estimate	λ, event frequency
Data required to estimate parameters	Number of events, x, in total time, t
Model elements	1. they involve a number of events that occurred, and an exposure time, or time

	at risk, when the events could have occurred
	2. they occur random events in time
	3. they initiate a quick response to repair the system to normal

It is standard to assume that the event count has a **Poisson distribution** [A]. The usual assumptions (following Thompson 1981) for a **Poisson process** [A] are [2]:

1. There is a rate $\lambda > 0$, such that for any **interval** [A] with short exposure time Δt the probability of an occurrence in the interval is approximately $\lambda \times \Delta t$. It implies that the rate λ does not change over time.

2. Exactly simultaneous events do not occur.

3. Occurrences of events in disjoint exposure time periods are statistically independent, which means that the past history does not affect the present.

Under the above assumptions, the number of occurrences X in some fixed exposure time t is a Poisson distributed random variable with **mean** [A] $\mu = \lambda t$, λ denotes the event frequency, with units events per unit time.

For any particular number x, the probability of x initiating events in time t is

$$\mathbf{Pr(X = x)} = \mathbf{e^{-\lambda t}(\lambda t)^x / x!} \qquad\qquad \textbf{(5-1)}$$

The emergency lighting systems failures satisfy the assumptions and elements of the initiating events model, so these failure data can be analyzed like initiating event data.

5.3   Frequentist and Bayesian inferences [A] [2]

There are two approaches to estimate parameters which are Bayesian approach and the frequentist, or classical, approach. The parameter estimated in this report is the failure rate of emergency lighting system, λ.

For Frequentist inference, the probability distributions are never used to describe parameters, because the parameters are not random. In another word, λ in this approach is fixed, not random. However, the data are random. Therefore, the **maximum likelihood estimator** [A] and the **confidence bounds** are all random.

In the Bayesian inference, the unknown parameter λ is quantified by an initial probability distribution, the prior distribution. Based on the data, the prior belief about the parameter is updated to the **posterior distribution** [A]. The final statement of this inference is quantifying the final uncertainty about the parameter based on

the posterior distribution.

When estimating parameters for PRA, the Bayesian method works better, for two reasons [2]:

First, data from reliable equipment are typically **sparse***(refer to "sparse" described in chapter 5.2), with few or even zero observed failures. In this case, the Bayesian approach provides a mechanism for incorporating other information as prior belief for those data. (This will be shown in **chapter 5.3.2**)

Second, the Bayesian framework allows straightforward propagation of basic event uncertainties through a logical model, to produce an uncertainty on the frequency of the undesirable end state. (This function of Bayesian approach will **not be expressed** in this report) The frequentist approach cannot handle such complicated propagation of uncertainties except by rough approximations.

However, frequentist method has its uses. The primary use of the frequentist is in preliminary examination of the data, to check the correctness of model assumptions, and to decide which model to use. Then Bayesian methods are used for estimating the parameters. (See **chapter 5.3.2.4**) In addition, the frequentist estimates are useful for rough approximate calculations because they are often simpler to calculate than Bayesian estimates. (See **chapter 5.3.1**)The comparison of Bayesian and frequentist methods in PRA is shown in **Table 5-4**

Table 5-4   Comparison of Bayesian and frequentist methods in PRA [2]

	Frequentist	Bayesian
Interpretation of probability	Long-term frequency after many hypothetical repetitions.	Measure of uncertainty, Quantification of degree of belief.
Unknown parameter	Constant, fixed	Constant, but assigned probability distribution, measuring current state of belief
Data	Random (before being observed)	Random for intermediate calculations. Fixed (after being observed) for the final conclusions.
Typical estimators	Maximum likelihood estimator (MLE), confidence interval.	Bayes posterior mean, credible interval
Interpretation of 90%	If many data sets are generated, 90% of the	We believe, and would give 9 to 1 odds in a wager, that the

interval for a parameter	resulting confidence intervals will contain the true parameter. We do not know if our interval is one of the unlucky ones. A confidence interval cannot be directly interpreted as a probability that the parameter lies in the interval.	parameter is in the interval. Because Bayesian probability intervals can be interpreted as probability statements about a parameter, they are easily combined with other sources of uncertainty in a PRA using the laws of probability.
Primary uses in PRA	1. Check model assumptions. 2. Provide quick estimates, without work of determining and justifying prior distribution.	1. Incorporate evidence from various sources, as prior distribution. 2. Propagate uncertainties through fault-tree and event-tree models.

5.3.1　Frequentist or classical estimation

5.3.1.1　Point estimate [A]

For point estimate, the most commonly used is the **Maximum likelihood estimate (MLE)**. It is found by taking the **probability distribution function (p.d.f)** [A], given by **Pr(X=x) = e$^{-\lambda t}$(λt)^x/x!**   (5-1) $\Pr(X = x) = e^{-\lambda t}(\lambda t)^x/x!$ (5-1, and treating it as a function of λ. The value of the parameter λ that maximum the probability is called the **MLE**. It can be calculated by taking the derivative of the **p.d.f**.

Taking the derivative of **Pr(X=x) = e$^{-\lambda t}$(λt)^x/x!**   (5-1), and setting it to 0, we get:

$$\frac{\partial \mathbf{Pr}}{\partial \lambda} = \frac{\partial}{\partial \lambda}\left(\frac{(\lambda t)^x}{e^{\lambda t}x!}\right) = \frac{x(\lambda t)^{x-1}te^{\lambda t}x! - (\lambda t)^x te^{\lambda t}x!}{(e^{\lambda t}x!)^2} = \mathbf{0} \qquad (5\text{-}2)$$

$$\hat{\lambda} = x/t \qquad (5\text{-}3)$$

*The hat notation is used to indicate that the **MLE** is an estimate calculated from the data, not the true, unknown λ.

5.3.1.2 Confidence interval for λ [A] [2]

The confidence interval is based on the **chi-squared** [A] (or in symbols, χ^2) distribution. $\chi_p^2(d)$ is the pth quantile, or (100p)th **percentile**[A], of the chi-squared distribution with d degrees of freedom.

For a $(1 - \alpha)$ confidence interval, or equivalently a $100(1 - \alpha)\%$ confidence interval, the lower limit is

$$\lambda_{conf.\alpha/2} = \frac{\chi_{\frac{\alpha}{2}}^2(2x)}{2t} \qquad (5\text{-}4)$$

If x = 0, this formula is undefined, but then simply set

$$\lambda_{conf.\alpha/2} = 0 \qquad (5\text{-}5)$$

Similarly, the upper limit is

$$\lambda_{conf.1-\alpha/2} = \frac{\chi_{1-\frac{\alpha}{2}}^2(2x+2)}{2t} \qquad (5\text{-}6)$$

These formulas are in terms of α. For example, set $\alpha=0.1$, which means that the formulas are given for a 90% confidence interval. These formulas involve the 5th percentile of a chi-squared distribution with 2x degrees of freedom, and the 95th percentile of a chi-squared distribution with (2x+2) degrees of freedom.

The resulting confidence interval is conservative in the sense that the actual confidence level is no smaller than the nominal level of $100(1 - \alpha)\%$, but it could be larger. This **conservatism** is inherent in confidence intervals based on discrete data.

In the frequentist approach, λ is fixed and the data are random. Therefore, the maximum likelihood estimator and the confidence limits are all random. For most data sets the MLE, $\hat{\lambda}$, will be close to the true value of λ, and the confidence interval will contain λ. However, sometimes the MLE will be rather far from λ, and sometimes the 90% confidence interval will not contain λ.

5.3.1.3 Data analysis

Take a set of the data of emergency lights stated above as an example.

Building number in X prefecture	Total number of testing lights	Number of lights that needed correction

1	31	1

As referred above, the total number of testing lights will be considered as the years that the testing period. Thus, we can transfer the set of data to that there is one light failed event in the last 31 years. Therefore, the estimated event rate for the building is

$$\hat{\lambda} = \frac{x}{t} = \frac{1}{31} = 0.0322581$$

with events per year.

And the 90% confidence limits are

$$\lambda_{conf.0.05} = \frac{\frac{\chi^2_{0.05}(2)}{2}}{2*31} = \frac{0.103}{62} = 0.00165$$

$$\lambda_{conf.0.95} = \frac{\chi^2_{0.95}(4)}{2*31} = \frac{9.488}{62} = 0.153$$

with events per year.

The calculated results of estimated event rate for each building and **90% confidence interval** are given in **Table 5-5** the Maximum likelihood estimates (MLEs) and 90% confidence intervals are plotted in **Figure 5-6**

Table 5-5 Maximum likelihood estimates (MLEs) and 90% confidence intervals

t_i	x_i	MLE	$\chi^2_{0.05}(2x)$	$\chi^2_{0.05}(2x)/2t$	$\chi^2_{0.95}(2x+2)$	$\chi^2_{0.95}(2x+2)/2t$
31	1	0.032258065	0.102586589	0.001654622	9.487729037	0.153027888
27	1	0.037037037	0.102586589	0.001899752	9.487729037	0.175698686
80	3	0.0375	1.635382894	0.010221143	15.50731306	0.096920707
25	1	0.04	0.102586589	0.002051732	9.487729037	0.189754581
150	6	0.04	5.226029488	0.017420098	23.6847913	0.078949304
24	1	0.041666667	0.102586589	0.002137221	9.487729037	0.197661022
21	1	0.047619048	0.102586589	0.002442538	9.487729037	0.22589831
20	1	0.05	0.102586589	0.002564665	9.487729037	0.237193226
20	1	0.05	0.102586589	0.002564665	9.487729037	0.237193226
19	1	0.052631579	0.102586589	0.002699647	9.487729037	0.24967708
38	2	0.052631579	0.710723021	0.009351619	12.59158724	0.16567878
32	2	0.0625	0.710723021	0.011105047	12.59158724	0.196743551
16	1	0.0625	0.102586589	0.003205831	9.487729037	0.296491532
15	1	0.066666667	0.102586589	0.003419553	9.487729037	0.316257635
30	2	0.066666667	0.710723021	0.011845384	12.59158724	0.209859787
15	1	0.066666667	0.102586589	0.003419553	9.487729037	0.316257635
15	1	0.066666667	0.102586589	0.003419553	9.487729037	0.316257635

14	1	0.071428571	0.102586589	0.003663807	9.487729037	0.338847466
14	1	0.071428571	0.102586589	0.003663807	9.487729037	0.338847466
27	2	0.074074074	0.710723021	0.013161537	12.59158724	0.233177542
13	1	0.076923077	0.102586589	0.003945638	9.487729037	0.364912655
13	1	0.076923077	0.102586589	0.003945638	9.487729037	0.364912655
13	1	0.076923077	0.102586589	0.003945638	9.487729037	0.364912655
38	3	0.078947368	1.635382894	0.021518196	15.50731306	0.204043593
12	1	0.083333333	0.102586589	0.004274441	9.487729037	0.395322043
12	1	0.083333333	0.102586589	0.004274441	9.487729037	0.395322043
12	1	0.083333333	0.102586589	0.004274441	9.487729037	0.395322043
12	1	0.083333333	0.102586589	0.004274441	9.487729037	0.395322043
155	13	0.083870968	15.37915658	0.049610183	41.33713815	0.133345607
23	2	0.086956522	0.710723021	0.0154505	12.59158724	0.273730157
22	2	0.090909091	0.710723021	0.016152796	12.59158724	0.286172437
11	1	0.090909091	0.102586589	0.004663027	9.487729037	0.431260411
11	1	0.090909091	0.102586589	0.004663027	9.487729037	0.431260411
10	1	0.1	0.102586589	0.005129329	9.487729037	0.474386452
10	1	0.1	0.102586589	0.005129329	9.487729037	0.474386452
10	1	0.1	0.102586589	0.005129329	9.487729037	0.474386452
29	3	0.103448276	1.635382894	0.028196257	15.50731306	0.267367466
38	4	0.105263158	2.732636793	0.035955747	18.30703805	0.24088208
19	2	0.105263158	0.710723021	0.018703237	12.59158724	0.331357559
9	1	0.111111111	0.102586589	0.005699255	9.487729037	0.527096058
9	1	0.111111111	0.102586589	0.005699255	9.487729037	0.527096058
9	1	0.111111111	0.102586589	0.005699255	9.487729037	0.527096058
9	1	0.111111111	0.102586589	0.005699255	9.487729037	0.527096058
9	1	0.111111111	0.102586589	0.005699255	9.487729037	0.527096058
9	1	0.111111111	0.102586589	0.005699255	9.487729037	0.527096058
44	5	0.113636364	3.940299136	0.044776127	21.02606982	0.238932612
8	1	0.125	0.102586589	0.006411662	9.487729037	0.592983065
16	2	0.125	0.710723021	0.022210094	12.59158724	0.393487101
23	3	0.130434783	1.635382894	0.035551802	15.50731306	0.337115501
23	3	0.130434783	1.635382894	0.035551802	15.50731306	0.337115501
15	2	0.133333333	0.710723021	0.023690767	12.59158724	0.419719575
15	2	0.133333333	0.710723021	0.023690767	12.59158724	0.419719575
15	2	0.133333333	0.710723021	0.023690767	12.59158724	0.419719575
7	1	0.142857143	0.102586589	0.007327613	9.487729037	0.677694931
14	2	0.142857143	0.710723021	0.025382965	12.59158724	0.449699544
14	2	0.142857143	0.710723021	0.025382965	12.59158724	0.449699544
7	1	0.142857143	0.102586589	0.007327613	9.487729037	0.677694931
28	4	0.142857143	2.732636793	0.048797086	18.30703805	0.326911394
7	1	0.142857143	0.102586589	0.007327613	9.487729037	0.677694931
13	2	0.153846154	0.710723021	0.027335501	12.59158724	0.484291817

13	2	0.153846154	0.710723021	0.027335501	12.59158724	0.484291817
18	3	0.166666667	1.635382894	0.045427303	15.50731306	0.430758696
17	3	0.176470588	1.635382894	0.048099497	15.50731306	0.456097443
17	3	0.176470588	1.635382894	0.048099497	15.50731306	0.456097443
17	3	0.176470588	1.635382894	0.048099497	15.50731306	0.456097443
17	3	0.176470588	1.635382894	0.048099497	15.50731306	0.456097443
22	4	0.181818182	2.732636793	0.062105382	18.30703805	0.416069047
11	2	0.181818182	0.710723021	0.032305592	12.59158724	0.572344875
37	7	0.189189189	6.570631384	0.088792316	26.2962276	0.355354427
5	1	0.2	0.102586589	0.010258659	9.487729037	0.948772904
10	2	0.2	0.710723021	0.035536151	12.59158724	0.629579362
9	2	0.222222222	0.710723021	0.039484612	12.59158724	0.699532625
13	3	0.230769231	1.635382894	0.062899342	15.50731306	0.596435118
82	19	0.231707317	24.88390438	0.151731124	55.75847928	0.339990727
8	2	0.25	0.710723021	0.044420189	12.59158724	0.786974203
8	2	0.25	0.710723021	0.044420189	12.59158724	0.786974203
8	2	0.25	0.710723021	0.044420189	12.59158724	0.786974203
8	2	0.25	0.710723021	0.044420189	12.59158724	0.786974203
20	5	0.25	3.940299136	0.098507478	21.02606982	0.525651745
8	2	0.25	0.710723021	0.044420189	12.59158724	0.786974203
12	3	0.25	1.635382894	0.068140954	15.50731306	0.646138044
19	5	0.263157895	3.940299136	0.103692083	21.02606982	0.553317627
15	4	0.266666667	2.732636793	0.091087893	18.30703805	0.610234602
15	4	0.266666667	2.732636793	0.091087893	18.30703805	0.610234602
11	3	0.272727273	1.635382894	0.074335586	15.50731306	0.704877866
11	3	0.272727273	1.635382894	0.074335586	15.50731306	0.704877866
11	3	0.272727273	1.635382894	0.074335586	15.50731306	0.704877866
10	3	0.3	1.635382894	0.081769145	15.50731306	0.775365653
10	3	0.3	1.635382894	0.081769145	15.50731306	0.775365653
19	6	0.315789474	5.226029488	0.137527092	23.6847913	0.623283982
6	2	0.333333333	0.710723021	0.059226918	12.59158724	1.049298937
9	3	0.333333333	1.635382894	0.090854605	15.50731306	0.861517392
9	3	0.333333333	1.635382894	0.090854605	15.50731306	0.861517392
18	6	0.333333333	5.226029488	0.145167486	23.6847913	0.65791087
3	1	0.333333333	0.102586589	0.017097765	9.487729037	1.581288173
14	5	0.357142857	3.940299136	0.140724969	21.02606982	0.750931065
11	4	0.363636364	2.732636793	0.124210763	18.30703805	0.832138093
10	4	0.4	2.732636793	0.13663184	18.30703805	0.915351903
15	6	0.4	5.226029488	0.174200983	23.6847913	0.789493043
14	6	0.428571429	5.226029488	0.18664391	23.6847913	0.845885404
9	4	0.444444444	2.732636793	0.151813155	18.30703805	1.01705767
4	2	0.5	0.710723021	0.088840378	12.59158724	1.573948405
2	1	0.5	0.102586589	0.025646647	9.487729037	2.371932259

10	5	0.5	3.940299136	0.197014957	21.02606982	1.051303491
17	11	0.647058824	12.33801458	0.362882782	36.4150285	1.07103025
3	2	0.666666667	0.710723021	0.118453837	12.59158724	2.098597874
13	9	0.692307692	9.390455081	0.361171349	31.41043284	1.208093571
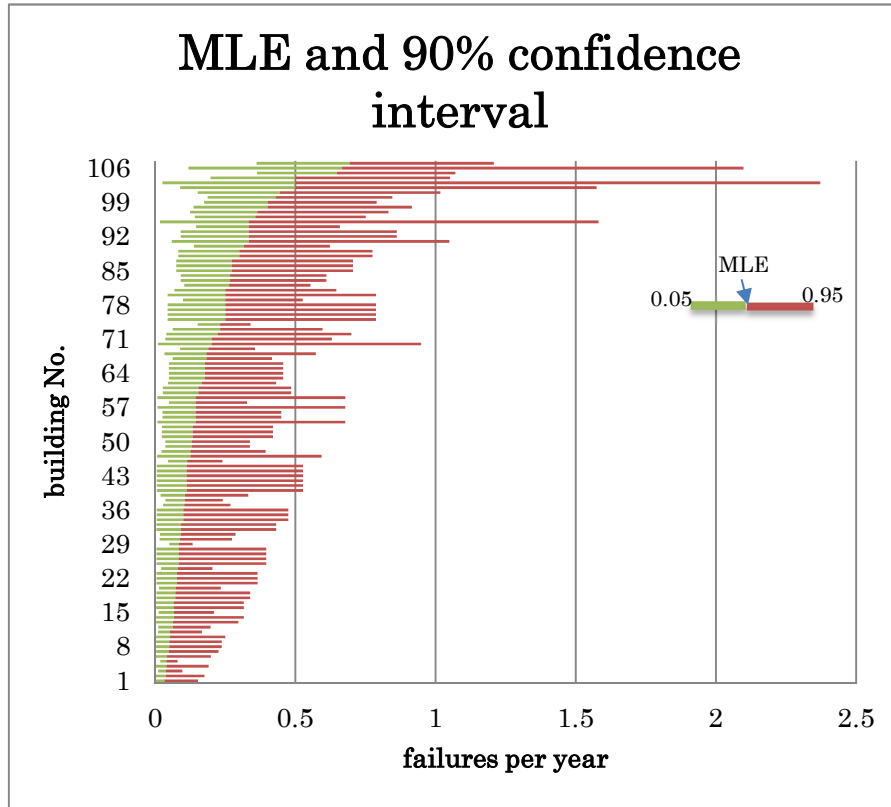
Figure 5-1 MLE and Confidence intervals for emergency lights data

5.3.2 Bayesian inference

5.3.2.1 Overview

Bayesian parameter estimation involves four steps. [2]

The first step is to identify the estimated parameter(s), which involves consideration of the assumed distribution of the data that will be collected. (Here is the failure rate λ). The second step is to develop a **prior distribution** that quantifies the unknown parameter(s) (The prior belief about λ). The third step is to collect the data sample, and to construct the **likelihood[A] function** (This is given by **equation Pr(X = x) = $e^{-\lambda} (\lambda t)^x / x!$** (5-1). It is written as a function of λ). The fourth and final step is to combine the prior distribution with the data sample using **Bayes' Theorem**

to construct the **posterior distribution**. In this case, this theorem says that

$$f_{post}(\lambda) \propto likelihood(\lambda) \times f_{prior}(\lambda)$$

The symbol '$\propto$' means 'is proportional to'.

5.3.2.2   Prior distribution [A]

There are several kinds of distributions that can be considered as prior distributions. The simplest prior distribution is discrete. The next simplest prior is **conjugate**[A]. The conjugate prior combines with the likelihood to give a posterior that can be evaluated by simple formulas. So I will estimate the parameter with a conjugate prior. And there are also several possible conjugate priors, such as **Informative priors, Noninformative prior** and so on.

5.3.2.3   Estimation with a conjugate prior

The conjugate family of prior distributions for Poisson data is the family of **gamma distributions** [A]. [2]

Probability density function (p.d.f) [A] for the two-parameter (α and β) gamma distribution:

$$\mathbf{f(\lambda) = \beta^{\alpha}[\Gamma(\alpha)]^{-1}\lambda^{\alpha-1}e^{-\beta\lambda}} \quad \mathbf{\lambda \geq 0, \alpha > 0, \beta > 0} \tag{5-7}$$

$$\mathbf{\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}dx} \tag{5-8}$$

Where $\Gamma(\alpha)$ is the gamma function and λ is the frequency of event per year, α is unitless, and is a shape parameter, β has units of year, and is a kind of scale parameter.

As stated earlier, the posterior distribution is related to the prior distribution by

$$f_{post}(\lambda) \propto \Pr(X = x|\lambda) \times f_{prior}(\lambda)$$

$\mathbf{=\beta\alpha[\Gamma(\alpha)]-1\lambda\alpha-1e-\beta\lambda}$ $\mathbf{\lambda\geq0,\ \alpha>0,\beta>0}$ (5-7), the gamma p.d.f can be expressed as follows

$$f(\lambda) \propto \lambda^{\alpha-1}e^{-\beta}$$

Combining the gamma distribution and the Poisson likelihood, the posterior

distribution will be expressed as

$$f_{post}(\lambda) \propto \frac{e^{-\lambda t}(\lambda t)^x}{x!} \times \lambda^{\alpha-1}e^{-\beta}$$

$$\propto \lambda^{(x+\alpha)-1}e^{-(t+\beta)\lambda}$$

This result also proves the meaning of conjugate that if the prior distribution is a member of the family (in this case, the gamma family), the posterior distribution is a member of the same family. The posterior distribution is

$$\mathbf{f}_{post}(\lambda) = \boldsymbol{\beta}_{post}{}^{\alpha_{post}}[\Gamma(\alpha_{post})]^{-1}\lambda^{\alpha_{post}-1}e^{-\beta_{post}\lambda} \quad \lambda \geq 0, \; \alpha_{post} > 0, \beta_{post} > \qquad (5\text{-}9)$$

Where

$$\boldsymbol{\alpha}_{post} = \boldsymbol{x} + \boldsymbol{\alpha}_{prior}, \qquad \boldsymbol{\beta}_{post} = \boldsymbol{t} + \boldsymbol{\beta}_{prior} \qquad (5\text{-}10)$$

The posterior mean is

$$\mathbf{E}(\lambda) = \boldsymbol{\alpha}_{post}/\boldsymbol{\beta}_{post} \qquad (5\text{-}11)$$

and the posterior variance is

$$Var(\lambda) = \boldsymbol{\alpha}_{post}/(\boldsymbol{\beta}_{post})^2 \qquad (5\text{-}12)$$

The percentiles of the gamma distribution can be found from a tabulation of the **chi-squared distribution**, possibly interpolating the table. The (100p)th is given by:

$$\lambda_p = \frac{\chi_p^2(2\alpha_{post})}{2\beta_{post}} \qquad (5\text{-}13)$$

Where $\chi_p^2(d)$ is the pth quantile, or(100p)th percentile, of a chi-squared distribution with d degrees of freedom.

Prior distributions are named 'prior' for a reason [2]: they reflect information that does not come from the current data. Ideally, generic data provide the basis for prior belief. However, as data limited, there is not generic data for the prior belief of the emergency lights. So I will try to deal with these existing data in two ways.

➢ First, try to conduct generic data for the prior belief based on these data, and then estimate with informative prior. (See chapter 5.3.2.4)

➢ Second, estimate them with noninformative prior. (See chapter 5.3.2.5)

5.3.2.4 Estimation with informative priors

With regard to the first handling way, the generic data here is a gamma distribution that can describe the variability of failure rate λ of the emergency lighting equipment across the building as a prior.   In order to conduct the generic data, I will regard the data recorded in **Table 5-2** as data from the same building roughly and then confirm that if they are fitting a gamma probability model or not. The way to fitting a gamma probability model is referring to the "Modeling Time to Recovery and Initiating Event Frequency for Loss of Off-site Power Incidents at Nuclear Power Plants" [3]

5.3.2.4.1 Data processing

As stated in **chapter 5.3.1** , one of uses of Frequentist approach is shown in here. The $\hat{\lambda}$ denotes the maximum likelihood estimate (MLE) of failure rate of the emergency lighting equipment, which are previously listed in **Table 5-5.** ' Freq' here means the frequency that each λ occurs, which result from the ratio of the number of the buildings that have same λ, **n** given by the data in **Table 5-5** to the total number of the target buildings, **107**. 'cum.Freq' means the cumulative density of λ. The meanings of parameters and the values of them are recorded in **Table 5-6**   and **Table 5-7.**

The relationship between the Failure rate of the emergency lighting equipment, λ and Cumulative density of λ, cum. Freq is plotted in **Figure 5-2.** And the corresponding graph of 1-cum.Freq is plotted in **Figure 5-3**

Table 5-6   Meanings of parameters

$\hat{\lambda}$	MLE of Failure rate of the emergency lighting equipment (Table 5-1)
Freq	The density of λ (the frequency that each λ occurs)(=P(Λ = λ))
cum. Freq	Cumulative density of λ (=P(Λ ≤ λ))

Table 5-7   Values of parameters

$\hat{\lambda}$	0.0323	0.0370	0.0375	0.0400	0.0417	0.0476	0.0500	0.0526	0.0625
n	1	1	1	2	1	1	2	2	2
Freq	0.0093	0.0093	0.0093	0.0187	0.0093	0.0093	0.0187	0.0187	0.0187
cum. Freq	0.0093	0.0187	0.0280	0.0467	0.0561	0.0654	0.0841	0.1028	0.1215
$\hat{\lambda}$	0.0667	0.0714	0.0741	0.0769	0.0789	0.0833	0.0839	0.0870	0.0909
n	4	2	1	3	1	4	1	1	3

Freq	0.0374	0.0187	0.0093	0.0280	0.0093	0.0374	0.0093	0.0093	0.0280
cum. Freq	0.1589	0.1776	0.1869	0.2150	0.2243	0.2617	0.2710	0.2804	0.3084

$\hat{\lambda}$	0.1000	0.1034	0.1053	0.1111	0.1136	0.1250	0.1304	0.1333	0.1429
n	3	1	2	6	1	2	2	3	6
Freq	0.0280	0.0093	0.0187	0.0561	0.0093	0.0187	0.0187	0.0280	0.0561
cum. Freq	0.3364	0.3458	0.3645	0.4206	0.4299	0.4486	0.4673	0.4953	0.5514

$\hat{\lambda}$	0.1538	0.1667	0.1765	0.1818	0.1892	0.2000	0.2222	0.2308	0.2317
n	2	1	4	2	1	2	1	1	1
Freq	0.0187	0.0093	0.0374	0.0187	0.0093	0.0187	0.0093	0.0093	0.0093
cum. Freq	0.5701	0.5794	0.6168	0.6355	0.6449	0.6636	0.6729	0.6822	0.6916

$\hat{\lambda}$	0.2500	0.2632	0.2667	0.2727	0.3000	0.3158	0.3333	0.3571	0.3636
n	7	1	2	3	2	1	5	1	1
Freq	0.0654	0.0093	0.0187	0.0280	0.0187	0.0093	0.0467	0.0093	0.0093
cum. Freq	0.7570	0.7664	0.7850	0.8131	0.8318	0.8411	0.8879	0.8972	0.9065

$\hat{\lambda}$	0.4000	0.4286	0.4444	0.5000	0.6471	0.6667	0.6923		
n	2	1	1	3	1	1	1		
Freq	0.0187	0.0093	0.0093	0.0280	0.0093	0.0093	0.0093		
cum. Freq	0.9252	0.9346	0.9439	0.9720	0.9813	0.9907	1.0000		
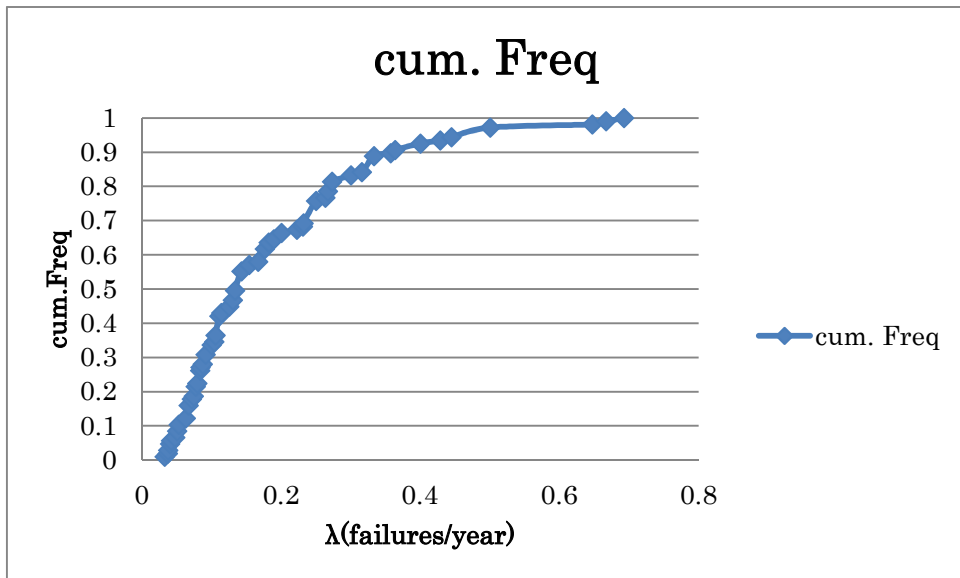
Figure 5-2　The relationship between the Failure rate of the emergency lighting equipment, λ and Cumulative density of λ, cum. Freq
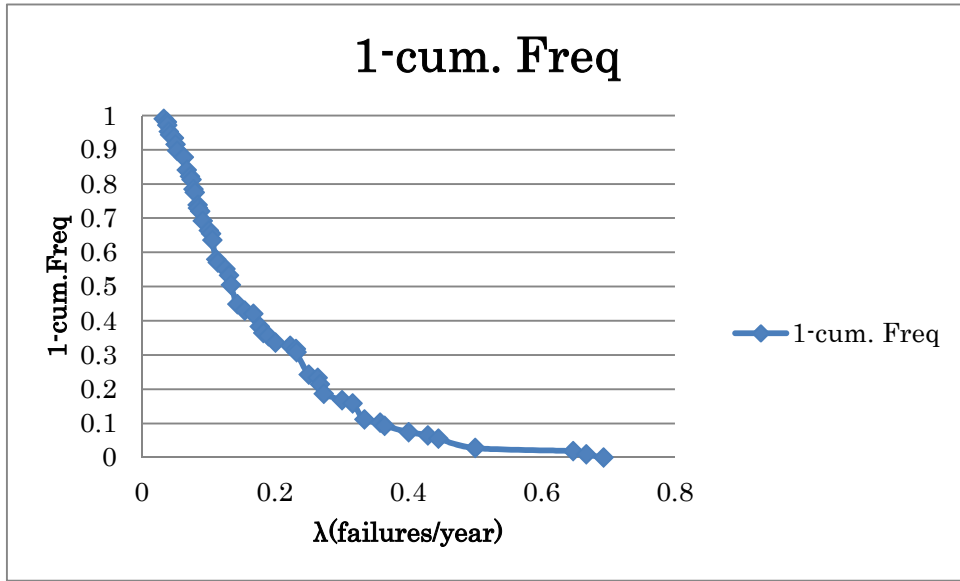
1-cum. Freq

Figure 5-3　The relationship between the Failure rate of the emergency lighting equipment, λ and 1-Cumulative density of λ, 1-cum. Freq

5.3.2.4.2　Fitting gamma probability models

Lawless (1982, pages 204-206) provides a discussion of the maximum likelihood procedure used for estimating the parameters α and β in the density function $=\beta\alpha/\Gamma(\alpha)]-1\lambda\alpha-1e-\beta\lambda$　$\lambda\geq0, \alpha>0, \beta>0$　(5-7). His procedure can be summarized using the following steps.[3]

1. Find the arithmetic mean $(\bar{\lambda})$ and the geometric mean $(\tilde{\lambda})$ for the sample data.

2. Calculate s=log $(\bar{\lambda}/\tilde{\lambda})$.

3. Compute $\hat{\alpha} \approx s^{-1}(17.79728 + 11.968477s + s^2)^{-1}$

$$* (8.898919 + 9.059950s + 0.9775373s^2) \ \ if \ 0.5772 < s \leq 17$$

Or $\hat{\alpha} \approx s^{-1}(0.5000876 + 0.1648852s - 0.0544274 + s^2) \ \ if \ 0 < s \leq 0.5772$

4. Compute $\hat{\beta} = \hat{\alpha}/\bar{\lambda}$

The detail calculations based on the values of λ in **Table 5-7** of the parameters α and β are as follows,

1. Arithmetic mean $(\bar{\lambda})$

$$\bar{\lambda} = \frac{0.032258 + 0.037037 + 0.0375 + \cdots + 0.692308}{52} = 0.20065$$

75

Geometric mean $(\tilde{\lambda})$

$$\tilde{\lambda} = \sqrt[52]{0.032258 \times 0.037037 \times 0.0375 \times \ldots \times 0.692308}$$

$$=0.14498$$

2. Calculate s

$$s=\log (\bar{\lambda}/\tilde{\lambda}) =\log_e \frac{0.20065}{0.14498} = 0.32497$$

3. Compute $\hat{\alpha}$

As $0 < s \leq 0.5772$

$$\hat{\alpha} \approx s^{-1}(0.5000876 + 0.1648852s - 0.0544274 + s^2)$$

$$= 1.68607$$

4. Compute $\hat{\beta}$

$$\hat{\beta} = \frac{\hat{\alpha}}{\bar{\lambda}} = \frac{1.68607}{0.20065} = 8.4032$$

The estimates of the parameters α, β can be used to obtain the estimated distribution function F(λ)=P(Λ ≤ λ)=$\int_0^\lambda f(\lambda)$ dλ., which gives the probability that the failure rate of the emergency lights are less than λ failures per year. However, since the less failure rates are expected, the P (Λ ≥ λ) is of interest in this application, that is the probability that the failure rate of the emergency lights will be larger than some failure rate λ failures per year. The results of **cumulative distribution function (c.d.f)** [A] **F(λ) and 1-F(λ)** calculated by Excel are recorded in **Table 5-8.** The relationship between the failure rate λ and the probability when Λ ≥ λ is shown in **Figure 5-4.** The graph of 1-F(λ) can be added to the graph of 1-cum.Freq in **Figure 5-3**.

The graphs of the fitted gamma probability models appear in **Figure 5-5**

Table 5-8   Cumulative distribution function (c.d.f) [A] F(λ) and 1-F(λ)

λ_i	0.032	0.037	0.038	0.040	0.042	0.048	0.050	0.053
F(λ_i)	0.061	0.075	0.077	0.085	0.090	0.109	0.117	0.126
1-F(λ_i)	0.939	0.925	0.923	0.915	0.910	0.891	0.883	0.874
λ_i	0.063	0.067	0.071	0.074	0.077	0.079	0.083	0.084
F(λ_i)	0.160	0.175	0.192	0.202	0.212	0.219	0.235	0.237
1-F(λ_i)	0.840	0.825	0.808	0.798	0.788	0.781	0.765	0.763
λ_i	0.087	0.091	0.100	0.103	0.105	0.111	0.114	0.125
F(λ_i)	0.248	0.262	0.295	0.307	0.313	0.334	0.343	0.381
1-F(λ_i)	0.752	0.738	0.705	0.693	0.687	0.666	0.657	0.619
λ_i	0.130	0.133	0.143	0.154	0.167	0.176	0.182	0.189
F(λ_i)	0.399	0.409	0.440	0.474	0.512	0.539	0.554	0.573
1-F(λ_i)	0.601	0.591	0.560	0.526	0.488	0.461	0.446	0.427
λ_i	0.200	0.222	0.231	0.232	0.250	0.263	0.267	0.273
F(λ_i)	0.601	0.653	0.671	0.673	0.709	0.733	0.739	0.749
1-F(λ_i)	0.399	0.347	0.329	0.327	0.291	0.267	0.261	0.251
λ_i	0.300	0.316	0.333	0.357	0.364	0.400	0.429	0.444
F(λ_i)	0.791	0.812	0.834	0.859	0.865	0.895	0.915	0.924
1-F(λ_i)	0.209	0.188	0.166	0.141	0.135	0.105	0.085	0.076
λ_i	0.500	0.647	0.667	0.692				
F(λ_i)	0.949	0.983	0.985	0.988				
1-F(λ_i)	0.051	0.017	0.015	0.012				
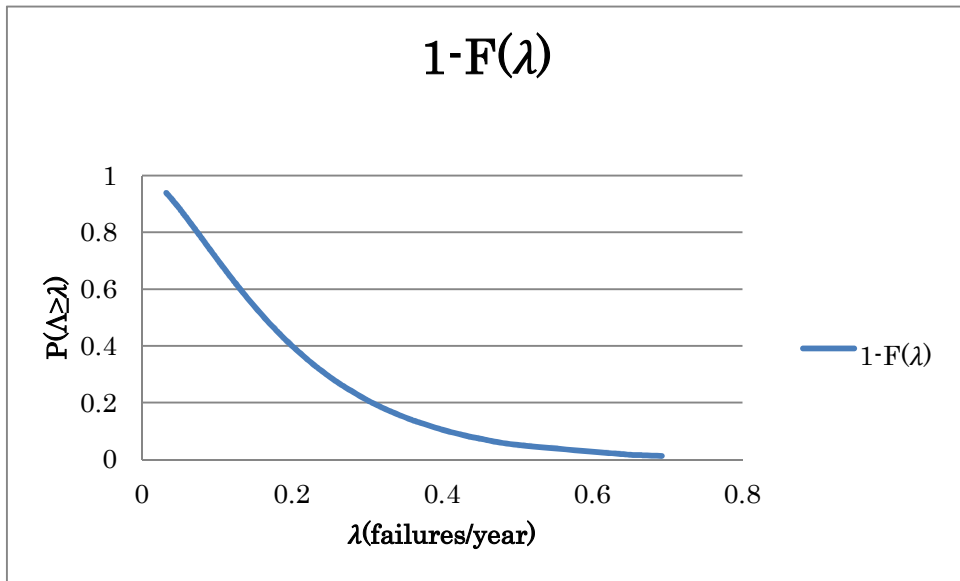
Figure 5-4   The relationship between the failure rate λ and the probability when $\Lambda \geq \lambda$
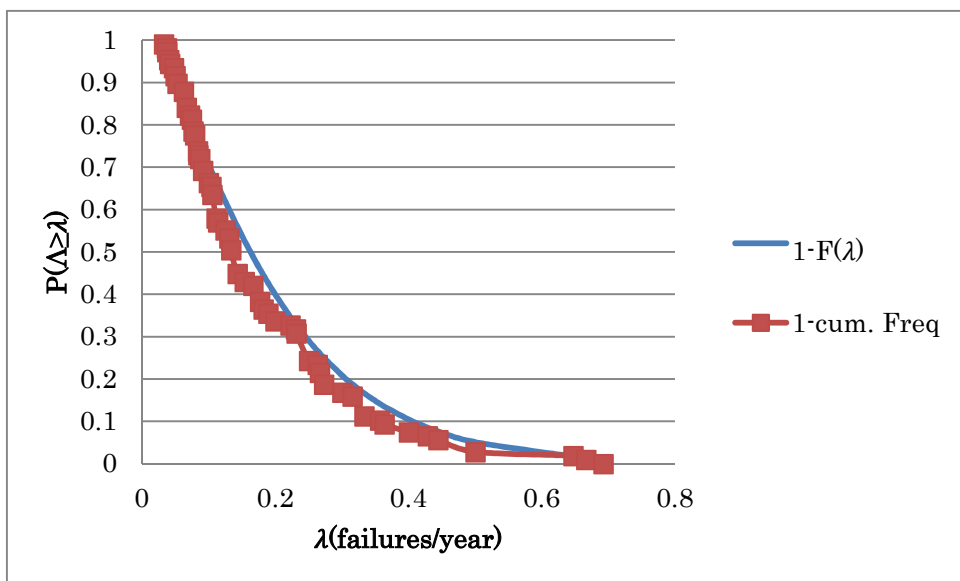
Figure 5-5   Gamma Probability Model for emergency lighting equipment inspection data

Figure 5-5 shows that the gamma probability model provides a good fit to the emergency lighting equipment inspection data. In this case, we can consider roughly that for the emergency lighting equipment, the variability of failure rate λ across the building can be described as a gamma distribution with shape parameter α =1.68607, and scale parameter β =8.4032 failures per year, though the data used here are from different buildings.

5.3.2.4.3 Data estimation

Therefore, we use the gamma distribution stated above as the prior.

Then, $\alpha_{prior}=$ 1.68607 $\qquad$ $\beta_{prior}=$ 8.4032

The update formulas are as follows given by

Where

$$\boldsymbol{\alpha_{post}} = \boldsymbol{x} + \boldsymbol{\alpha_{prior}}, \qquad\qquad \boldsymbol{\beta_{post}} = \boldsymbol{t} + \boldsymbol{\beta_{prior}} \qquad\qquad (5\text{-}10):$$

$\alpha_{post} = x_i + \alpha_{prior}=x_i + 1.68607 \quad \beta_{post} = t_i + \beta_{prior}=t_i + 8.4032$

And the **90% credible interval** [A] can be conducted as

$$\lambda_{0.05} = \frac{\chi^2_{0.05}(2\alpha_{post})}{2\beta_{post}} = \frac{\chi^2_{0.05}[2(x_i + 1.68607)]}{2(t_i + 8.4032)}$$

$$\lambda_{0.95} = \frac{\chi^2_{0.95}(2\alpha_{post})}{2\beta_{post}} = \frac{\chi^2_{0.05}[2(x_i + 1.68607)]}{2(t_i + 8.4032)}$$

given by **equation** $\boldsymbol{\lambda p}= \frac{\chi^2_p(2\alpha_{post})}{2\beta_{post}}$ $\qquad\qquad$ (5-13)

The α_{post}, β_{post} and the mean of the posterior distributions, as well as **90% credible interval** of failure rate of emergency lighting equipment, λ_i of each target building are given in **Table 5-9**. The posterior mean and 90% credible interval are plotted in **Figure 5-6.**

Table 5-9　Posterior mean and 90% credible interval of failure rate

α_{prior}	1.68607							
β_{prior}	8.4032							
t_i	x_i	α_{post} $= x_i$ $+ \alpha_{prior}$	β_{post} $= t_i$ $+ \beta_{prior}$	Posterior mean	$\chi^2_{0.05}(2\alpha_{post})$	$\frac{\chi^2_{0.05}(2\alpha_{post})}{2\beta_{post}}$	$\chi^2_{0.95}(2\alpha_{post})$	$\frac{\chi^2_{0.95}(2\alpha_{post})}{2\beta_{post}}$
31	1	2.68607	39.40320	0.06817	1.14548	0.01454	11.07050	0.14048
27	1	2.68607	35.40320	0.07587	1.14548	0.01618	11.07050	0.15635
80	3	4.68607	88.40320	0.05301	3.32511	0.01881	16.91898	0.09569

25	1	2.68607	33.40320	0.08041	1.14548	0.01715	11.07050	0.16571
150	6	7.68607	158.40320	0.04852	7.26094	0.02292	24.99579	0.07890
24	1	2.68607	32.40320	0.08290	1.14548	0.01768	11.07050	0.17082
21	1	2.68607	29.40320	0.09135	1.14548	0.01948	11.07050	0.18825
20	1	2.68607	28.40320	0.09457	1.14548	0.02016	11.07050	0.19488
20	1	2.68607	28.40320	0.09457	1.14548	0.02016	11.07050	0.19488
19	1	2.68607	27.40320	0.09802	1.14548	0.02090	11.07050	0.20199
38	2	3.68607	46.40320	0.07944	2.16735	0.02335	14.06714	0.15158
32	2	3.68607	40.40320	0.09123	2.16735	0.02682	14.06714	0.17408
16	1	2.68607	24.40320	0.11007	1.14548	0.02347	11.07050	0.22682
15	1	2.68607	23.40320	0.11477	1.14548	0.02447	11.07050	0.23652
30	2	3.68607	38.40320	0.09598	2.16735	0.02822	14.06714	0.18315
15	1	2.68607	23.40320	0.11477	1.14548	0.02447	11.07050	0.23652
15	1	2.68607	23.40320	0.11477	1.14548	0.02447	11.07050	0.23652
14	1	2.68607	22.40320	0.11990	1.14548	0.02557	11.07050	0.24707
14	1	2.68607	22.40320	0.11990	1.14548	0.02557	11.07050	0.24707
27	2	3.68607	35.40320	0.10412	2.16735	0.03061	14.06714	0.19867
13	1	2.68607	21.40320	0.12550	1.14548	0.02676	11.07050	0.25862
13	1	2.68607	21.40320	0.12550	1.14548	0.02676	11.07050	0.25862
13	1	2.68607	21.40320	0.12550	1.14548	0.02676	11.07050	0.25862
38	3	4.68607	46.40320	0.10099	3.32511	0.03583	16.91898	0.18230
12	1	2.68607	20.40320	0.13165	1.14548	0.02807	11.07050	0.27129

		7	0					
12	1	2.68607	20.40320	0.13165	1.14548	0.02807	11.07050	0.27129
12	1	2.68607	20.40320	0.13165	1.14548	0.02807	11.07050	0.27129
12	1	2.68607	20.40320	0.13165	1.14548	0.02807	11.07050	0.27129
15 5	1 3	14.68607	163.40320	0.08988	17.70837	0.05419	42.55697	0.13022
23	2	3.68607	31.40320	0.11738	2.16735	0.03451	14.06714	0.22398
22	2	3.68607	30.40320	0.12124	2.16735	0.03564	14.06714	0.23134
11	1	2.68607	19.40320	0.13843	1.14548	0.02952	11.07050	0.28528
11	1	2.68607	19.40320	0.13843	1.14548	0.02952	11.07050	0.28528
10	1	2.68607	18.40320	0.14596	1.14548	0.03112	11.07050	0.30078
10	1	2.68607	18.40320	0.14596	1.14548	0.03112	11.07050	0.30078
10	1	2.68607	18.40320	0.14596	1.14548	0.03112	11.07050	0.30078
29	3	4.68607	37.40320	0.12529	3.32511	0.04445	16.91898	0.22617
38	4	5.68607	46.40320	0.12254	4.57481	0.04929	19.67514	0.21200
19	2	3.68607	27.40320	0.13451	2.16735	0.03955	14.06714	0.25667
9	1	2.68607	17.40320	0.15434	1.14548	0.03291	11.07050	0.31806
9	1	2.68607	17.40320	0.15434	1.14548	0.03291	11.07050	0.31806
9	1	2.68607	17.40320	0.15434	1.14548	0.03291	11.07050	0.31806
9	1	2.68607	17.40320	0.15434	1.14548	0.03291	11.07050	0.31806
9	1	2.68607	17.40320	0.15434	1.14548	0.03291	11.07050	0.31806
9	1	2.68607	17.40320	0.15434	1.14548	0.03291	11.07050	0.31806
44	5	6.68607	52.40320	0.12759	5.89186	0.05622	22.36203	0.21337

8	1	2.68607	16.40320	0.16375	1.14548	0.03492	11.07050	0.33745
16	2	3.68607	24.40320	0.15105	2.16735	0.04441	14.06714	0.28822
23	3	4.68607	31.40320	0.14922	3.32511	0.05294	16.91898	0.26938
23	3	4.68607	31.40320	0.14922	3.32511	0.05294	16.91898	0.26938
15	2	3.68607	23.40320	0.15750	2.16735	0.04630	14.06714	0.30054
15	2	3.68607	23.40320	0.15750	2.16735	0.04630	14.06714	0.30054
15	2	3.68607	23.40320	0.15750	2.16735	0.04630	14.06714	0.30054
7	1	2.68607	15.40320	0.17438	1.14548	0.03718	11.07050	0.35936
14	2	3.68607	22.40320	0.16453	2.16735	0.04837	14.06714	0.31395
14	2	3.68607	22.40320	0.16453	2.16735	0.04837	14.06714	0.31395
7	1	2.68607	15.40320	0.17438	1.14548	0.03718	11.07050	0.35936
28	4	5.68607	36.40320	0.15620	4.57481	0.06284	19.67514	0.27024
7	1	2.68607	15.40320	0.17438	1.14548	0.03718	11.07050	0.35936
13	2	3.68607	21.40320	0.17222	2.16735	0.05063	14.06714	0.32862
13	2	3.68607	21.40320	0.17222	2.16735	0.05063	14.06714	0.32862
18	3	4.68607	26.40320	0.17748	3.32511	0.06297	16.91898	0.32040
17	3	4.68607	25.40320	0.18447	3.32511	0.06545	16.91898	0.33301
17	3	4.68607	25.40320	0.18447	3.32511	0.06545	16.91898	0.33301
17	3	4.68607	25.40320	0.18447	3.32511	0.06545	16.91898	0.33301
17	3	4.68607	25.40320	0.18447	3.32511	0.06545	16.91898	0.33301
22	4	5.68607	30.40320	0.18702	4.57481	0.07524	19.67514	0.32357
11	2	3.6860	19.4032	0.18997	2.16735	0.05585	14.06714	0.36250

		7	0					
37	7	8.68607	45.40320	0.19131	8.67176	0.09550	27.58711	0.30380
5	1	2.68607	13.40320	0.20041	1.14548	0.04273	11.07050	0.41298
10	2	3.68607	18.40320	0.20030	2.16735	0.05889	14.06714	0.38219
9	2	3.68607	17.40320	0.21180	2.16735	0.06227	14.06714	0.40415
13	3	4.68607	21.40320	0.21894	3.32511	0.07768	16.91898	0.39524
82	19	20.68607	90.40320	0.22882	27.32555	0.15113	56.94239	0.31494
8	2	3.68607	16.40320	0.22472	2.16735	0.06606	14.06714	0.42879
8	2	3.68607	16.40320	0.22472	2.16735	0.06606	14.06714	0.42879
8	2	3.68607	16.40320	0.22472	2.16735	0.06606	14.06714	0.42879
8	2	3.68607	16.40320	0.22472	2.16735	0.06606	14.06714	0.42879
20	5	6.68607	28.40320	0.23540	5.89186	0.10372	22.36203	0.39365
8	2	3.68607	16.40320	0.22472	2.16735	0.06606	14.06714	0.42879
12	3	4.68607	20.40320	0.22967	3.32511	0.08149	16.91898	0.41462
19	5	6.68607	27.40320	0.24399	5.89186	0.10750	22.36203	0.40802
15	4	5.68607	23.40320	0.24296	4.57481	0.09774	19.67514	0.42035
15	4	5.68607	23.40320	0.24296	4.57481	0.09774	19.67514	0.42035
11	3	4.68607	19.40320	0.24151	3.32511	0.08568	16.91898	0.43598
11	3	4.68607	19.40320	0.24151	3.32511	0.08568	16.91898	0.43598
11	3	4.68607	19.40320	0.24151	3.32511	0.08568	16.91898	0.43598
10	3	4.68607	18.40320	0.25463	3.32511	0.09034	16.91898	0.45967
10	3	4.68607	18.40320	0.25463	3.32511	0.09034	16.91898	0.45967

19	6	7.68607	27.40320	0.28048	7.26094	0.13248	24.99579	0.45607
6	2	3.68607	14.40320	0.25592	2.16735	0.07524	14.06714	0.48833
9	3	4.68607	17.40320	0.26926	3.32511	0.09553	16.91898	0.48609
9	3	4.68607	17.40320	0.26926	3.32511	0.09553	16.91898	0.48609
18	6	7.68607	26.40320	0.29110	7.26094	0.13750	24.99579	0.47335
3	1	2.68607	11.40320	0.23555	1.14548	0.05023	11.07050	0.48541
14	5	6.68607	22.40320	0.29844	5.89186	0.13150	22.36203	0.49908
11	4	5.68607	19.40320	0.29305	4.57481	0.11789	19.67514	0.50701
10	4	5.68607	18.40320	0.30897	4.57481	0.12429	19.67514	0.53456
15	6	7.68607	23.40320	0.32842	7.26094	0.15513	24.99579	0.53403
14	6	7.68607	22.40320	0.34308	7.26094	0.16205	24.99579	0.55786
9	4	5.68607	17.40320	0.32673	4.57481	0.13144	19.67514	0.56527
4	2	3.68607	12.40320	0.29719	2.16735	0.08737	14.06714	0.56708
2	1	2.68607	10.40320	0.25820	1.14548	0.05505	11.07050	0.53207
10	5	6.68607	18.40320	0.36331	5.89186	0.16008	22.36203	0.60756
17	11	12.68607	25.40320	0.49939	14.61141	0.28759	37.65248	0.74110
3	2	3.68607	11.40320	0.32325	2.16735	0.09503	14.06714	0.61681
13	9	10.68607	21.40320	0.49927	11.59131	0.27078	32.67057	0.76322
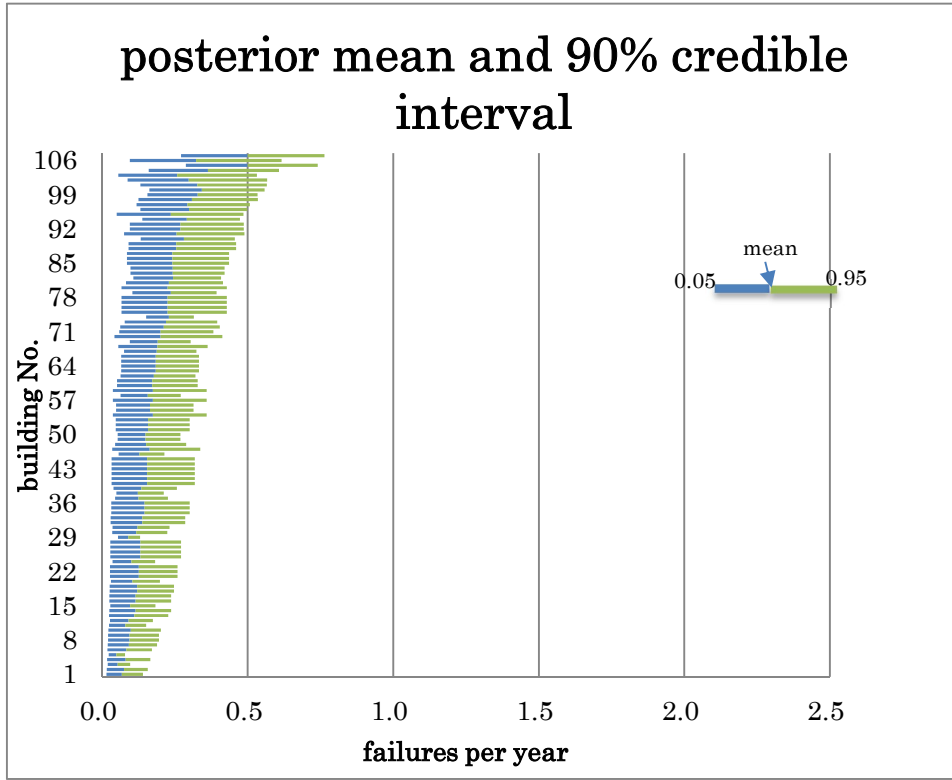
Figure 5-6　Posterior mean and 90% credible interval with informative prior

5.3.2.5　Estimation with noninformative prior

The **Jeffreys noninformative prior** is intended to convey little prior belief or information, thus allowing the data to speak for themselves. This is useful when no informed consensus exists about the true value of the unknown parameter. It is also useful when the prior distribution may be challenged by people with various agendas.

With Poisson data, the Jeffreys noninformative prior is obtained if the shape parameter of a gamma distribution is taken to be $\alpha = 1/2$ and the parameter β is taken to be zero. (See, for example, Box and Tiao 1973.) [2]

For the data that consist of x events in time t, formal application of the update formulas yields

$$\alpha_{post} = x_i + 1/2 \qquad \beta_{post} = t_i + 0 \qquad\qquad (5\text{-}14)$$

That is, the Bayes posterior distribution for λ_i is gamma $(x_i+1/2, t_i)$.

The 90% posterior credible limits are

$$\lambda_{0.05} = \frac{\chi^2_{0.05}(2x+1)}{2t} \qquad \lambda_{0.95} = \frac{\chi^2_{0.95}(2x+1)}{2t} \tag{5-15}$$

We can find that the equations of the 90% posterior credible interval are similar to the equations of the confidence interval given in equation $\lambda_{conf.\alpha/2} = \frac{\chi^2_{\frac{\alpha}{2}}(2x)}{2t}$ (5-4) and equation $\lambda_{conf.1-\alpha/2} = \frac{\chi^2_{1-\frac{\alpha}{2}}(2x+2)}{2t}$ (5-6):

$$\lambda_{conf.\alpha/2} = \frac{\chi^2_{\frac{\alpha}{2}}(2x)}{2t} \quad , \quad \lambda_{conf.1-\alpha/2} = \frac{\chi^2_{1-\frac{\alpha}{2}}(2x+2)}{2t}.$$

The confidence intervals differ from the Bayes credible intervals only in the degrees of freedom. The lower and upper confidence limits have degrees of freedom 2x and 2x+2, respectively. The two Bayesian limits each use the average, 2x + 1. The confidence interval is wider than the Jeffreys credible interval, a reflection of the **conservatism of confidence limits with discrete data**. However, the similarity between the confidence limits and the Jeffreys limits shows that the result using the Jeffreys prior will resemble the result using frequentist approach, that is, using no prior information at all. [2]

Consider again the data in Table 5-2, with x_i events in t_i years, and use the Jeffreys noninformative prior. The α_{post}, β_{post} and the mean of the posterior distributions, as well as **90% Bayes credible interval** of failure rate of emergency lighting equipment, λ_i of each target building are given in **Table 5-10**. The posterior mean and 90% Bayes credible interval are plotted in **Figure 5-7**.

Table 5-10　Posterior mean and 90% credible interval

α_{prior}	1/2							
β_{prior}	0							
t_i	x_i	α_{post} $= x_i$ $+ 0.5$	β_{post} $= t_i$ $+ 0$	Posterior mean	$\chi^2_{0.05}(2x_i + 1)$	$\dfrac{\chi^2_{0.05}(2x_i + 1)}{2t_i}$	$\chi^2_{0.95}(2x_i + 1)$	$\dfrac{\chi^2_{0.95}(2x_i + 1)}{2t_i}$
31	1	1.5	31	0.04839	0.35185	0.00567	7.81473	0.12604
27	1	1.5	27	0.05556	0.35185	0.00652	7.81473	0.14472
80	3	3.5	80	0.04375	2.16735	0.01355	14.06714	0.08792
25	1	1.5	25	0.06000	0.35185	0.00704	7.81473	0.15629
150	6	6.5	150	0.04333	5.89186	0.01964	22.36203	0.07454
24	1	1.5	24	0.06250	0.35185	0.00733	7.81473	0.16281
21	1	1.5	21	0.07143	0.35185	0.00838	7.81473	0.18606
20	1	1.5	20	0.07500	0.35185	0.00880	7.81473	0.19537
20	1	1.5	20	0.07500	0.35185	0.00880	7.81473	0.19537
19	1	1.5	19	0.07895	0.35185	0.00926	7.81473	0.20565

38	2	2.5	38	0.06579	1.14548	0.01507	11.07050	0.14566
32	2	2.5	32	0.07813	1.14548	0.01790	11.07050	0.17298
16	1	1.5	16	0.09375	0.35185	0.01100	7.81473	0.24421
15	1	1.5	15	0.10000	0.35185	0.01173	7.81473	0.26049
30	2	2.5	30	0.08333	1.14548	0.01909	11.07050	0.18451
15	1	1.5	15	0.10000	0.35185	0.01173	7.81473	0.26049
15	1	1.5	15	0.10000	0.35185	0.01173	7.81473	0.26049
14	1	1.5	14	0.10714	0.35185	0.01257	7.81473	0.27910
14	1	1.5	14	0.10714	0.35185	0.01257	7.81473	0.27910
27	2	2.5	27	0.09259	1.14548	0.02121	11.07050	0.20501
13	1	1.5	13	0.11538	0.35185	0.01353	7.81473	0.30057
13	1	1.5	13	0.11538	0.35185	0.01353	7.81473	0.30057
13	1	1.5	13	0.11538	0.35185	0.01353	7.81473	0.30057
38	3	3.5	38	0.09211	2.16735	0.02852	14.06714	0.18509
12	1	1.5	12	0.12500	0.35185	0.01466	7.81473	0.32561
12	1	1.5	12	0.12500	0.35185	0.01466	7.81473	0.32561
12	1	1.5	12	0.12500	0.35185	0.01466	7.81473	0.32561
12	1	1.5	12	0.12500	0.35185	0.01466	7.81473	0.32561
155	13	13.5	155	0.08710	16.15140	0.05210	40.11327	0.12940
23	2	2.5	23	0.10870	1.14548	0.02490	11.07050	0.24066
22	2	2.5	22	0.11364	1.14548	0.02603	11.07050	0.25160
11	1	1.5	11	0.13636	0.35185	0.01599	7.81473	0.35521
11	1	1.5	11	0.13636	0.35185	0.01599	7.81473	0.35521
10	1	1.5	10	0.15000	0.35185	0.01759	7.81473	0.39074
10	1	1.5	10	0.15000	0.35185	0.01759	7.81473	0.39074
10	1	1.5	10	0.15000	0.35185	0.01759	7.81473	0.39074
29	3	3.5	29	0.12069	2.16735	0.03737	14.06714	0.24254
38	4	4.5	38	0.11842	3.32511	0.04375	16.91898	0.22262
19	2	2.5	19	0.13158	1.14548	0.03014	11.07050	0.29133
9	1	1.5	9	0.16667	0.35185	0.01955	7.81473	0.43415
9	1	1.5	9	0.16667	0.35185	0.01955	7.81473	0.43415
9	1	1.5	9	0.16667	0.35185	0.01955	7.81473	0.43415
9	1	1.5	9	0.16667	0.35185	0.01955	7.81473	0.43415
9	1	1.5	9	0.16667	0.35185	0.01955	7.81473	0.43415
9	1	1.5	9	0.16667	0.35185	0.01955	7.81473	0.43415
44	5	5.5	44	0.12500	4.57481	0.05199	19.67514	0.22358
8	1	1.5	8	0.18750	0.35185	0.02199	7.81473	0.48842
16	2	2.5	16	0.15625	1.14548	0.03580	11.07050	0.34595
23	3	3.5	23	0.15217	2.16735	0.04712	14.06714	0.30581
23	3	3.5	23	0.15217	2.16735	0.04712	14.06714	0.30581
15	2	2.5	15	0.16667	1.14548	0.03818	11.07050	0.36902
15	2	2.5	15	0.16667	1.14548	0.03818	11.07050	0.36902

15	2	2.5	15	0.16667	1.14548	0.03818	11.07050	0.36902
7	1	1.5	7	0.21429	0.35185	0.02513	7.81473	0.55819
14	2	2.5	14	0.17857	1.14548	0.04091	11.07050	0.39537
14	2	2.5	14	0.17857	1.14548	0.04091	11.07050	0.39537
7	1	1.5	7	0.21429	0.35185	0.02513	7.81473	0.55819
28	4	4.5	28	0.16071	3.32511	0.05938	16.91898	0.30212
7	1	1.5	7	0.21429	0.35185	0.02513	7.81473	0.55819
13	2	2.5	13	0.19231	1.14548	0.04406	11.07050	0.42579
13	2	2.5	13	0.19231	1.14548	0.04406	11.07050	0.42579
18	3	3.5	18	0.19444	2.16735	0.06020	14.06714	0.39075
17	3	3.5	17	0.20588	2.16735	0.06375	14.06714	0.41374
17	3	3.5	17	0.20588	2.16735	0.06375	14.06714	0.41374
17	3	3.5	17	0.20588	2.16735	0.06375	14.06714	0.41374
17	3	3.5	17	0.20588	2.16735	0.06375	14.06714	0.41374
22	4	4.5	22	0.20455	3.32511	0.07557	16.91898	0.38452
11	2	2.5	11	0.22727	1.14548	0.05207	11.07050	0.50320
37	7	7.5	37	0.20270	7.26094	0.09812	24.99579	0.33778
5	1	1.5	5	0.30000	0.35185	0.03518	7.81473	0.78147
10	2	2.5	10	0.25000	1.14548	0.05727	11.07050	0.55352
9	2	2.5	9	0.27778	1.14548	0.06364	11.07050	0.61503
13	3	3.5	13	0.26923	2.16735	0.08336	14.06714	0.54104
82	19	19.5	82	0.23780	25.69539	0.15668	54.57223	0.33276
8	2	2.5	8	0.31250	1.14548	0.07159	11.07050	0.69191
8	2	2.5	8	0.31250	1.14548	0.07159	11.07050	0.69191
8	2	2.5	8	0.31250	1.14548	0.07159	11.07050	0.69191
8	2	2.5	8	0.31250	1.14548	0.07159	11.07050	0.69191
20	5	5.5	20	0.27500	4.57481	0.11437	19.67514	0.49188
8	2	2.5	8	0.31250	1.14548	0.07159	11.07050	0.69191
12	3	3.5	12	0.29167	2.16735	0.09031	14.06714	0.58613
19	5	5.5	19	0.28947	4.57481	0.12039	19.67514	0.51777
15	4	4.5	15	0.30000	3.32511	0.11084	16.91898	0.56397
15	4	4.5	15	0.30000	3.32511	0.11084	16.91898	0.56397
11	3	3.5	11	0.31818	2.16735	0.09852	14.06714	0.63942
11	3	3.5	11	0.31818	2.16735	0.09852	14.06714	0.63942
11	3	3.5	11	0.31818	2.16735	0.09852	14.06714	0.63942
10	3	3.5	10	0.35000	2.16735	0.10837	14.06714	0.70336
10	3	3.5	10	0.35000	2.16735	0.10837	14.06714	0.70336
19	6	6.5	19	0.34211	5.89186	0.15505	22.36203	0.58847
6	2	2.5	6	0.41667	1.14548	0.09546	11.07050	0.92254
9	3	3.5	9	0.38889	2.16735	0.12041	14.06714	0.78151
9	3	3.5	9	0.38889	2.16735	0.12041	14.06714	0.78151
18	6	6.5	18	0.36111	5.89186	0.16366	22.36203	0.62117

|								
3	1	1.5	3	0.50000	0.35185	0.05864	7.81473	1.30245
14	5	5.5	14	0.39286	4.57481	0.16339	19.67514	0.70268
11	4	4.5	11	0.40909	3.32511	0.15114	16.91898	0.76904
10	4	4.5	10	0.45000	3.32511	0.16626	16.91898	0.84595
15	6	6.5	15	0.43333	5.89186	0.19640	22.36203	0.74540
14	6	6.5	14	0.46429	5.89186	0.21042	22.36203	0.79864
9	4	4.5	9	0.50000	3.32511	0.18473	16.91898	0.93994
4	2	2.5	4	0.62500	1.14548	0.14318	11.07050	1.38381
2	1	1.5	2	0.75000	0.35185	0.08796	7.81473	1.95368
10	5	5.5	10	0.55000	4.57481	0.22874	19.67514	0.98376
17	11	11.5	17	0.67647	13.09051	0.38502	35.17246	1.03448
3	2	2.5	3	0.83333	1.14548	0.19091	11.07050	1.84508
13	9	9.5	13	0.73077	10.11701	0.38912	30.14353	1.15937
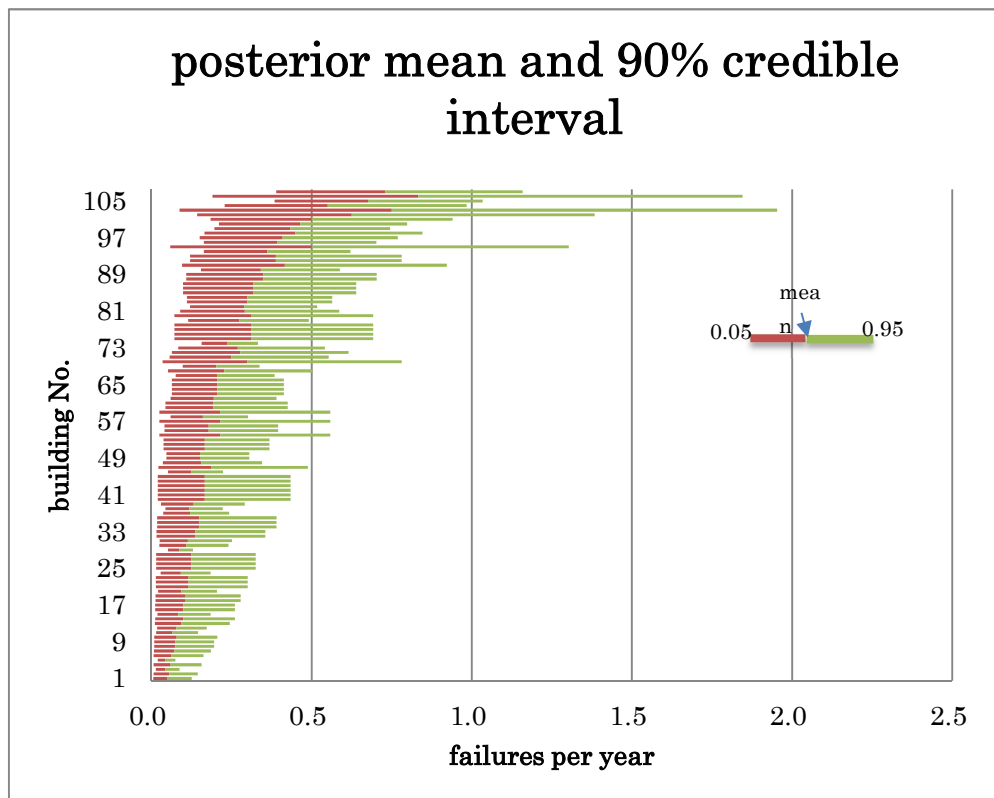
Figure 5-7 Posterior mean and 90% credible interval with Jeffreys noninformative prior

5.3.3 Comparison of estimates with the data of emergency lighting equipment using frequentist and Bayesian methods

Both the Frequentist approach and the Bayesian approach which are estimated with informative prior and noninformative prior are used to estimate the failure rate in each target building and to quantify the uncertainty about the failure rate, as the results shown in **Figure 5-1**, **Figure 5-6** and **Figure 5-7.** In other words, the MLE or the mean indicates the estimating failure rate of the emergency lighting equipment in each building; however, they are not the true failure rate, so the 90% confidence interval indicates that it has 90% probability of containing the true failure rate. 90% Bayesian credible interval has the same intuitive purpose as frequentist confidence intervals, but its definition and interpretation are different.

However, not all the approaches are proper for using in this case, thus comparisons should be conducted to discuss their features, merits and drawbacks and so on, and then choose a more suitable approach for the estimating of the emergency lighting equipment in buildings.

5.3.3.1 Frequentist VS Bayesian with Jeffreys noninformative prior

First of all, compare the results of the Frequentist approach and the Bayesian approach with Jeffreys noninformative prior, **Figure 5-1** and **Figure 5-7.** From these two figures we can see that the posterior credible intervals which result from the Jeffreys noninformative prior are numerically similar to the confidence intervals which result from the Frequentist approach because of the similarity between the confidence limits and the Jeffreys limits as noted in **Chapter 5.3.2.5.**

From these two figures we may find slight range differences between these two kinds of intervals, but not obviously. So I calculate the ranges of posterior credible intervals, "PCI" and ranges of confidence intervals, "CI" and then list them in **Table 5-11.** "No." represents the target building number. They show obviously that the posterior credible intervals are slightly shorter than the confidence intervals, as noted in **Chapter 5.3.2.5.**

To sum up, the point estimation by the MLE in frequentist approach is simper in form than the Bayes estimates. Meanwhile, the uncertainties of these estimates expressed by confidence intervals and credible intervals respectively have numerical similarity. However, the posterior credible intervals are shorter than the confidence intervals which means that the uncertainty about the failure rates estimated by the Bayesian with Jeffreys noninformative prior is smaller than that estimated by Frequentist. So comparing to the Frequentist approach, the Bayesian approach with Jeffreys noninformative prior is better for failure rate estimating.
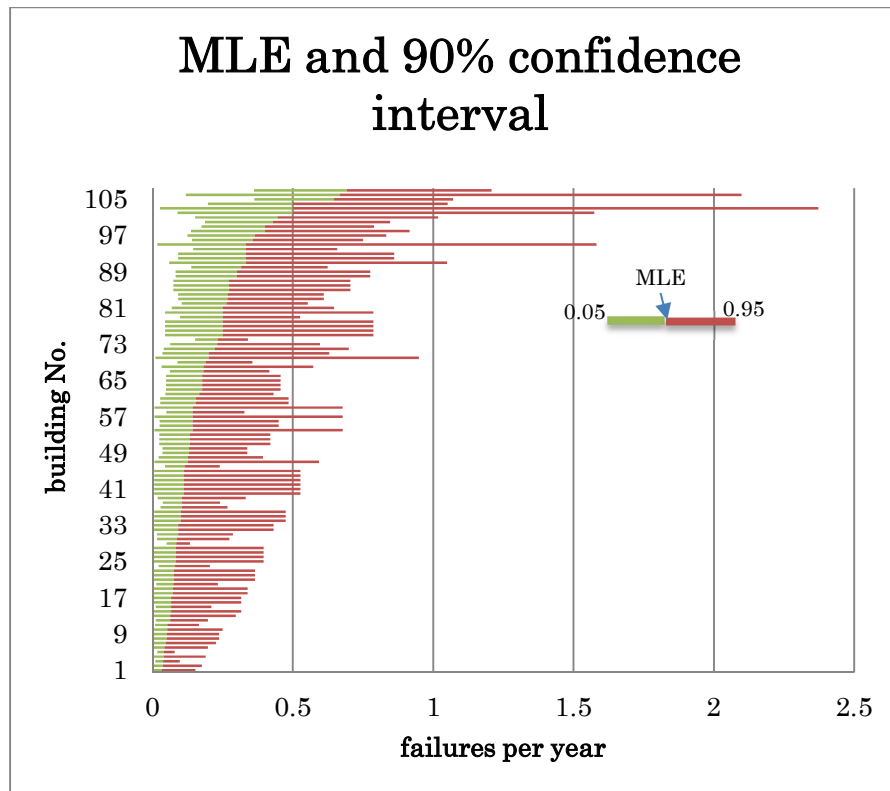
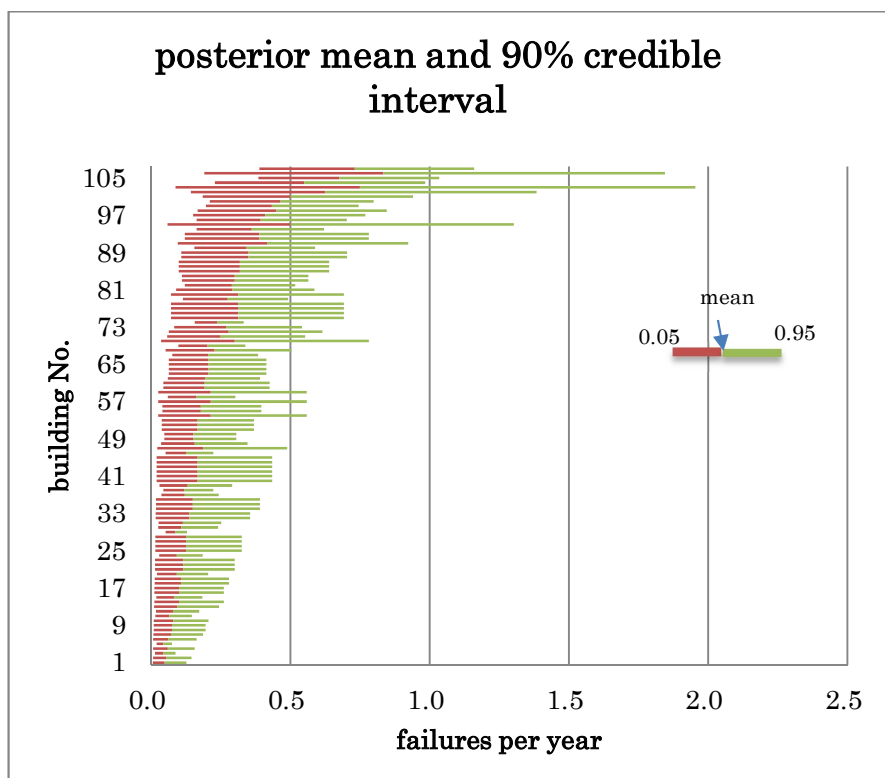**Figure 5-1 MLE and Confidence intervals for emergency lights data**

Figure 5-7 Posterior mean and 90% credible interval with Jeffreys noninformative prior

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Table 5-11　Ranges of posterior credible intervals and confidence intervals

No.	1	2	3	4	5	6	7	8	9	10
PCI	0.1204	0.1382	0.0744	0.1493	0.0549	0.1555	0.1777	0.1866	0.1866	0.1964
CI	0.1514	0.1738	0.0867	0.1877	0.0615	0.1955	0.2235	0.2346	0.2346	0.2470
PCI-CI	-0.0310	-0.0356	-0.0123	-0.0384	-0.0066	-0.0400	-0.0458	-0.0481	-0.0481	-0.0506

No.	11	12	13	14	15	16	17	18	19	20
PCI	0.1306	0.1551	0.2332	0.2488	0.1654	0.2488	0.2488	0.2665	0.2665	0.1838
CI	0.1563	0.1856	0.2933	0.3128	0.1980	0.3128	0.3128	0.3352	0.3352	0.2200
PCI-CI	-0.0257	-0.0306	-0.0601	-0.0641	-0.0326	-0.0641	-0.0641	-0.0687	-0.0687	-0.0362

No.	21	22	23	24	25	26	27	28	29	30
PCI	0.2870	0.2870	0.2870	0.1566	0.3110	0.3110	0.3110	0.3110	0.0773	0.2158
CI	0.3610	0.3610	0.3610	0.1825	0.3910	0.3910	0.3910	0.3910	0.0837	0.2583
PCI-CI	-0.0739	-0.0739	-0.0739	-0.0259	-0.0801	-0.0801	-0.0801	-0.0801	-0.0064	-0.0425

No.	31	32	33	34	35	36	37	38	39	40
PCI	0.2256	0.3392	0.3392	0.3731	0.3731	0.3731	0.2052	0.1789	0.2612	0.4146
CI	0.2700	0.4266	0.4266	0.4693	0.4693	0.4693	0.2392	0.2049	0.3127	0.5214
PCI-CI	-0.0445	-0.0874	-0.0874	-0.0961	-0.0961	-0.0961	-0.0340	-0.0261	-0.0515	-0.1068

No.	41	42	43	44	45	46	47	48	49	50
PCI	0.4146	0.4146	0.4146	0.4146	0.4146	0.1716	0.4664	0.3102	0.2587	0.2587
CI	0.5214	0.5214	0.5214	0.5214	0.5214	0.1942	0.5866	0.3713	0.3016	0.3016
PCI-CI	-0.1068	-0.1068	-0.1068	-0.1068	-0.1068	-0.0226	-0.1201	-0.0611	-0.0429	-0.0429

No.	51	52	53	54	55	56	57	58	59	60
PCI	0.3308	0.3308	0.3308	0.5331	0.3545	0.3545	0.5331	0.2427	0.5331	0.3817
CI	0.3960	0.3960	0.3960	0.6704	0.4243	0.4243	0.6704	0.2781	0.6704	0.4570
PCI-CI	-0.0652	-0.0652	-0.0652	-0.1373	-0.0699	-0.0699	-0.1373	-0.0354	-0.1373	-0.0752

No.	61	62	63	64	65	66	67	68	69	70
PCI	0.3817	0.3305	0.3500	0.3500	0.3500	0.3500	0.3090	0.4511	0.2397	0.7463
CI	0.4570	0.3853	0.4080	0.4080	0.4080	0.4080	0.3540	0.5400	0.2666	0.9385
PCI-CI	-0.0752	-0.0548	-0.0580	-0.0580	-0.0580	-0.0580	-0.0450	-0.0889	-0.0269	-0.1922

No.	71	72	73	74	75	76	77	78	79	80
PCI	0.4963	0.5514	0.4577	0.1761	0.6203	0.6203	0.6203	0.6203	0.3775	0.6203
CI	0.5940	0.6600	0.5335	0.1883	0.7426	0.7426	0.7426	0.7426	0.4271	0.7426
PCI-CI	-0.0978	-0.1087	-0.0759	-0.0122	-0.1222	-0.1222	-0.1222	-0.1222	-0.0496	-0.1222

No.	81	82	83	84	85	86	87	88	89	90
PCI	0.4958	0.3974	0.4531	0.4531	0.5409	0.5409	0.5409	0.5950	0.5950	0.4334
CI	0.5780	0.4496	0.5191	0.5191	0.6305	0.6305	0.6305	0.6936	0.6936	0.4858
PCI-CI	-0.0822	-0.0522	-0.0660	-0.0660	-0.0896	-0.0896	-0.0896	-0.0986	-0.0986	-0.0523

No.	91	92	93	94	95	96	97	98	99	100
PCI	0.8271	0.6611	0.6611	0.4575	1.2438	0.5393	0.6179	0.6797	0.5490	0.5882
CI	0.9901	0.7707	0.7707	0.5127	1.5642	0.6102	0.7079	0.7787	0.6153	0.6592
PCI-CI	-0.1630	-0.1096	-0.1096	-0.0552	-0.3204	-0.0709	-0.0900	-0.0990	-0.0663	-0.0710

No.	101	102	103	104	105	106	107			
PCI	0.7552	1.2406	1.8657	0.7550	0.6495	1.6542	0.7703			
CI	0.8652	1.4851	2.3463	0.8543	0.7081	1.9801	0.8469			
PCI-CI	-0.1100	-0.2445	-0.4806	-0.0993	-0.0587	-0.3260	-0.0767			

5.3.3.2 Bayesian with Jeffreys noninformative prior VS Bayesian with informative prior
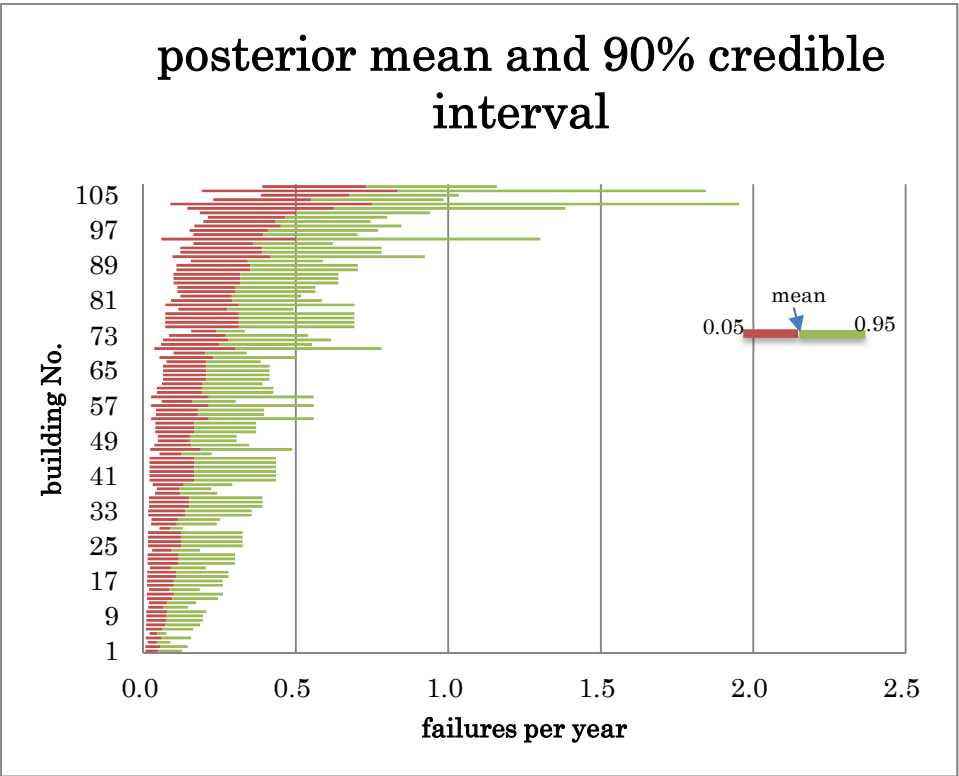

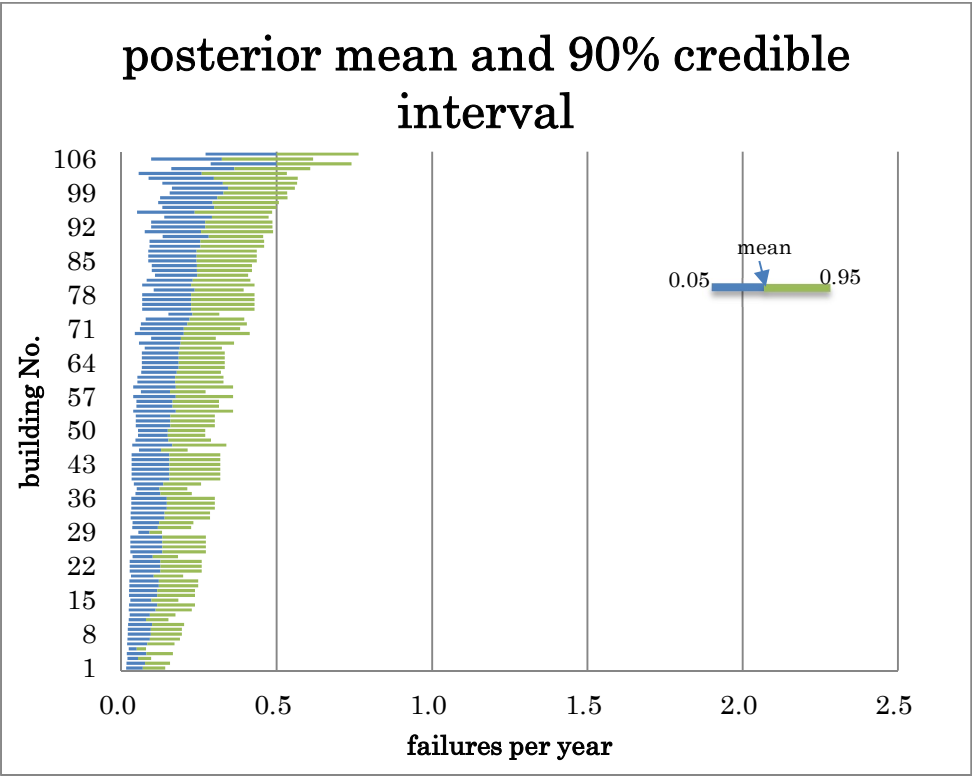Figure 5-7   Posterior mean and 90% credible interval with Jeffreys noninformative prior

Figure 5-6   Posterior mean and 90% credible interval with informative prior

Next, compare the results of the Bayesian approach with Jeffreys noninformative prior and with informative prior, **Figure 5-1** and **Figure 5-7.** From these two figures it seems that the posterior credible intervals which result from the informative prior are shorter than that resulting from the noninformative prior as a whole. To show it visually, I list the ranges of two kinds of credible intervals, "noninfo" and "info" of each building in **Table 5-12**. "No." represents the target building number. Almost all of them show smaller credible intervals which result from the informative prior, except the buildings which have relatively smaller failure rate.

Table 5-12 Credible interval ranges resulting from noninformative and informative prior

No.	1	2	3	4	5	6	7	8	9	10
noninfo	0.1204	0.1382	0.0744	0.1493	0.0549	0.1555	0.1777	0.1866	0.1866	0.1964
Info	0.1259	0.1402	0.0769	0.1486	0.0560	0.1531	0.1688	0.1747	0.1747	0.1811
Info-Noninfo	0.0056	0.0020	0.0025	-0.0007	0.0011	-0.0023	-0.0089	-0.0119	-0.0119	-0.0153
No.	11	12	13	14	15	16	17	18	19	20
noninfo	0.1306	0.1551	0.2332	0.2488	0.1654	0.2488	0.2488	0.2665	0.2665	0.1838
Info	0.1282	0.1473	0.2034	0.2120	0.1549	0.2120	0.2120	0.2215	0.2215	0.1681
Info-Noninfo	-0.0024	-0.0078	-0.0299	-0.0367	-0.0105	-0.0367	-0.0367	-0.0450	-0.0450	-0.0157
No.	21	22	23	24	25	26	27	28	29	30
noninfo	0.2870	0.2870	0.2870	0.1566	0.3110	0.3110	0.3110	0.3110	0.0773	0.2158
Info	0.2319	0.2319	0.2319	0.1465	0.2432	0.2432	0.2432	0.2432	0.0760	0.1895
Info-Noninfo	-0.0552	-0.0552	-0.0552	-0.0101	-0.0677	-0.0677	-0.0677	-0.0677	-0.0013	-0.0263
No.	31	32	33	34	35	36	37	38	39	40
noninfo	0.2256	0.3392	0.3392	0.3731	0.3731	0.3731	0.2052	0.1789	0.2612	0.4146
info	0.1957	0.2558	0.2558	0.2697	0.2697	0.2697	0.1817	0.1627	0.2171	0.2851
Info-Noninfo	-0.0299	-0.0835	-0.0835	-0.1035	-0.1035	-0.1035	-0.0234	-0.0162	-0.0441	-0.1295
No.	41	42	43	44	45	46	47	48	49	50
noninfo	0.4146	0.4146	0.4146	0.4146	0.4146	0.1716	0.4664	0.3102	0.2587	0.2587
Info	0.2851	0.2851	0.2851	0.2851	0.2851	0.1571	0.3025	0.2438	0.2164	0.2164
Info-Noninfo	-0.1295	-0.1295	-0.1295	-0.1295	-0.1295	-0.0144	-0.1639	-0.0663	-0.0423	-0.0423
No.	51	52	53	54	55	56	57	58	59	60
Noninfo	0.3308	0.3308	0.3308	0.5331	0.3545	0.3545	0.5331	0.2427	0.5331	0.3817
Info	0.2542	0.2542	0.2542	0.3222	0.2656	0.2656	0.3222	0.2074	0.3222	0.2780
Info-Noninfo	-0.0766	-0.0766	-0.0766	-0.2109	-0.0889	-0.0889	-0.2109	-0.0353	-0.2109	-0.1037
No.	61	62	63	64	65	66	67	68	69	70

Noninfo	0.3817	0.3305	0.3500	0.3500	0.3500	0.3500	0.3090	0.4511	0.2397	0.7463
Info	0.2780	0.2574	0.2676	0.2676	0.2676	0.2676	0.2483	0.3066	0.2083	0.3702
Info-Noninfo	-0.1037	-0.0731	-0.0824	-0.0824	-0.0824	-0.0824	-0.0606	-0.1445	-0.0314	-0.3760
No.	71	72	73	74	75	76	77	78	79	80
noninfo	0.4963	0.5514	0.4577	0.1761	0.6203	0.6203	0.6203	0.6203	0.3775	0.6203
info	0.3233	0.3419	0.3176	0.1638	0.3627	0.3627	0.3627	0.3627	0.2899	0.3627
Info-Noninfo	-0.1729	-0.2095	-0.1401	-0.0123	-0.2576	-0.2576	-0.2576	-0.2576	-0.0876	-0.2576
No.	81	82	83	84	85	86	87	88	89	90
noninfo	0.4958	0.3974	0.4531	0.4531	0.5409	0.5409	0.5409	0.5950	0.5950	0.4334
info	0.3331	0.3005	0.3226	0.3226	0.3503	0.3503	0.3503	0.3693	0.3693	0.3236
Info-Noninfo	-0.1627	-0.0969	-0.1305	-0.1305	-0.1906	-0.1906	-0.1906	-0.2257	-0.2257	-0.1098
No.	91	92	93	94	95	96	97	98	99	100
noninfo	0.8271	0.6611	0.6611	0.4575	1.2438	0.5393	0.6179	0.6797	0.5490	0.5882
info	0.4131	0.3906	0.3906	0.3358	0.4352	0.3676	0.3891	0.4103	0.3789	0.3958
Info-Noninfo	-0.4140	-0.2705	-0.2705	-0.1217	-0.8086	-0.1717	-0.2288	-0.2694	-0.1701	-0.1924
No.	101	102	103	104	105	106	107			
noninfo	0.7552	1.2406	1.8657	0.7550	0.6495	1.6542	0.7703			
info	0.4338	0.4797	0.4770	0.4475	0.4535	0.5218	0.4924			
Info-Noninfo	-0.3214	-0.7609	-1.3887	-0.3075	-0.1960	-1.1324	-0.2778			

In addition, I plot the prior mean and posterior mean with the informative prior and MLE in **Figure 5-8.** It shows that the posterior mean is between the prior mean and the MLE, if the prior mean exists (the prior mean of Bayes with Jeffreys noninformative prior is undefined). [2]
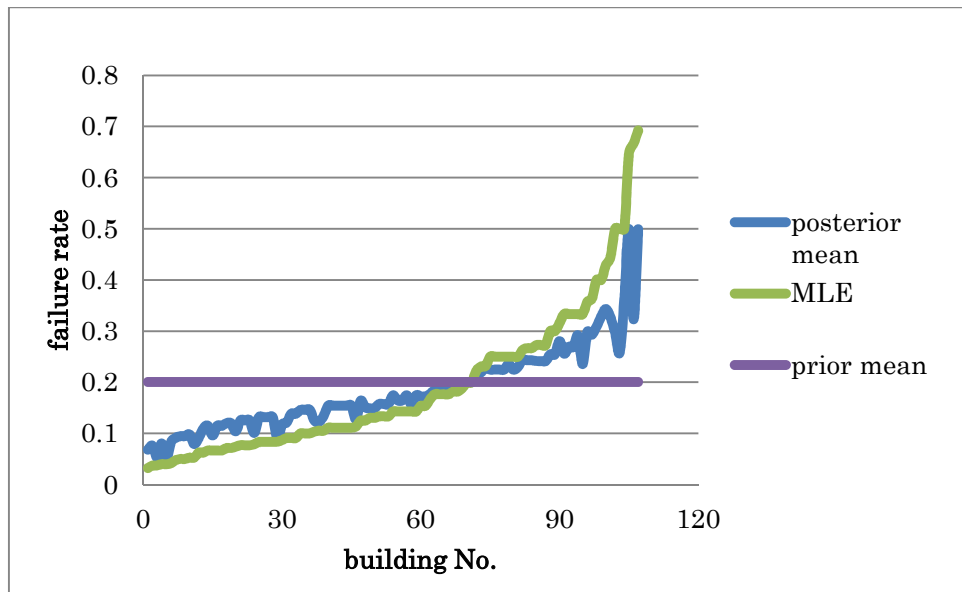
Figure 5-8　Prior mean and posterior mean with the informative prior and MLE

In summary, the uncertainty about the failure rates estimated by the Bayes with informative prior is smaller than that estimated by the Bayes with Jeffreys noninformative prior. Besides, if there is a generic data that could be as a prior, before making an exact calculation of the posterior mean, we could quickly determine a range that the posterior mean is belong to, which is based on the MLE and prior mean. Therefore, the Bayes with informative prior may be a more proper approach for failure rate estimating, with an essential prerequisite that the generic distribution is exist.

5.4 Appendix

A.   Probability Distributions [2]

A.1   The Poisson Distribution

The Poisson distribution provides a discrete probability model that is appropriate for many random phenomena that involve counts. A common use of the Poisson distribution is to describe the behavior of many rare event occurrences. The Poisson distribution is also frequently used in applications to describe the occurrence of system or component failures under steady-state conditions. These applications utilize the relationship between the Poisson and exponential (continuous random variable) distributions: the times between successive events follow an exponential distribution.

A process that leads to a   Poisson random variable is said to be a **Poisson process** [A].

The Poisson distribution describes the total number of events occurring in some interval of time t (or space). The p.d.f. of a Poisson random variable X, with parameter $\mu = \lambda t$, is

$$\Pr(X = x) = \mathbf{e^{-\lambda t}(\mu)^x/x!} = \mathbf{e^{-\lambda t}(\lambda t)^x/x!} \qquad\qquad \text{A-1}$$

For x=0, 1, 2, ..., and x! =x(x- 1)(x- 2) ...(2)(1)

The Poisson distribution has a single parameter μ, denoted Poisson (μ). If X denotes the number of events that occur during some time period of length t, then X is often assumed to have a Poisson distribution with parameter $\mu = \lambda t$. The variable λ is also referred to as the **event rate** (or **failure rate** when the events are failures). Note that λ has unit 1/time; thus, $\mu (= \lambda t)$ is dimensionless.

The expected number of events occurring in the interval 0 to t is $\mu = \lambda t$. Thus, the mean of the Poisson distribution is equal to the parameter of the distribution, which is why μ is often used to represent the parameter. The variance of the Poisson distribution is also equal to the parameter of the distribution. Therefore, for a Poisson (μ) random variable X, $E(X) = Var(X) = \mu = \lambda t$.

A.2   The Exponential Distribution

The exponential distribution is widely used for modeling time to failure and is inherently associated with the Poisson process [see Martz and Waller (1991)]. For a Poisson random variable X defining the number of failures in a time interval t and for a random variable T defining the time to failure, it can be shown that T has the exponential p.d.f.

$$f(t) = \lambda e^{-\lambda t}$$

for t > 0. Thus, the time to first failure and the times between successive failures follow an exponential distribution and the number of failures in a fixed time interval follows a Poisson distribution.

The exponential distribution parameter, λ, corresponds to the λt parameterization of **e**$-\boldsymbol{\lambda t(\lambda t)x/x!}$ and is referred to as the failure rate if the component or system is repaired and restarted immediately after each failure.

The c.d.f. of the exponential distribution is

$$F(t) = 1 - e^{-\lambda t}.$$

The exponential distribution with parameter λ is denoted exponential (λ). The mean and variance of an exponential (λ) distribution are

$$E(T) = 1/\lambda$$

$$Var(T) = 1/\lambda^2$$

A.3   The Gamma and Chi-Squared Distribution

The gamma distribution is an extension of the exponential distribution. It is

1. Sometimes used as a failure time model (Martz and Waller 1991).

2. And also often used as a prior distribution in **Bayesian estimation** of the failure rate parameter λ from Poisson (λt) or exponential (λ) data in PRA work.

The chi-squared distribution is a re-expression of a special case of the gamma distribution.

The distribution of the sum of independent exponential (λ) random variables is gamma, which forms the basis for a confidence interval for λ from exponential (λ) data. Because the sum of n independent exponentially distributed random variables has a gamma distribution, the gamma distribution is often used as the distribution of the time, or waiting time, to the nth event in a Poisson process.

For a random variable, T, that has a gamma distribution, the p.d.f. is

$$f(t) = \beta^\alpha [\Gamma(\alpha)]^{-1} t^{\alpha-1} e^{-\beta t}$$

for t, α, and $\beta > 0$.

Here

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

is the gamma function evaluated at α.

For Gamma Function $\Gamma(\alpha)$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx = \int_0^\infty \frac{1}{\alpha} e^{-x} dx^\alpha = \frac{1}{\alpha} \left(e^{-x} x^\alpha |_0^\infty + \int_0^\infty x^\alpha e^{-x} dx \right)$$

$$= \frac{1}{\alpha} \Gamma(\alpha+1)$$

$$\Rightarrow \alpha \Gamma(\alpha) = \Gamma(\alpha+1)$$

If α is a positive integer,

$$\Gamma(\alpha) = (\alpha-1)!$$

If α is a positive half-integers (for example 1/2, 3/2, 5/2), take n as a non-negative

integer, the function values are given by [6]

$$\Gamma(\alpha) = \Gamma\left(\frac{1}{2} + n\right) \approx \sqrt{\pi} \frac{(2n-1)!!}{2^n}$$

where n!! denotes the double factorial and, when n= 0, n!! = 1

The mean and variance of the gamma (α, β) random variable, T, are:

$$E(T) = \alpha/\beta$$

$$Var(T) = \alpha/\beta^2$$

The parameters α and β are referred to as the shape and scale parameters. When $\alpha <$ 1, the density becomes infinite at 0. When $\alpha= 1$, the gamma distribution reduces to an exponential (β^{-1}) distribution. When α is large, the distribution is approximately a normal distribution. **Figure 4** shows gamma densities with four shape parameters α. Also, the gamma ($\alpha= n/2$, $\beta= 1/2$) distribution is known as the chi-squared distribution with n degrees of freedom, denoted $\chi^2(n)$.

In addition, if T has a gamma (α, β) distribution, then $2\beta T$ has a $\chi^2(2\alpha)$ distribution, which forms the defining relationship between the two distributions. The gamma and chi-squared distributions can, therefore, be viewed as two ways of expressing one distribution. The $100 \times p$ percentile of a gamma (α, β) distribution is $\chi_p^2(2\alpha)/(2\beta)$, where $\chi_p^2(2\alpha)$ denotes the $100 \times p$ percentile of the chi-squared distribution with 2α degrees of freedom.

An alternative parameterization of the gamma distribution uses the scale parameter, say $\tau = \beta^{-1}$. If T has a gamma (α, τ) distribution, its p.d.f. is

$$f(t) = [\tau^\alpha \Gamma(\alpha)]^{-1} t^{\alpha-1} e^{-t/\tau}$$

for t, a, and r > 0. The mean and variance of the gamma (α, τ) random variable, T, are:

$$E(T) = \alpha\tau$$

$$Var(T) = \alpha\tau^2$$

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Note) this alternative parameterization is used in Excel (GAMMA.DIST 関数). The parameter "β" is the "τ" here.
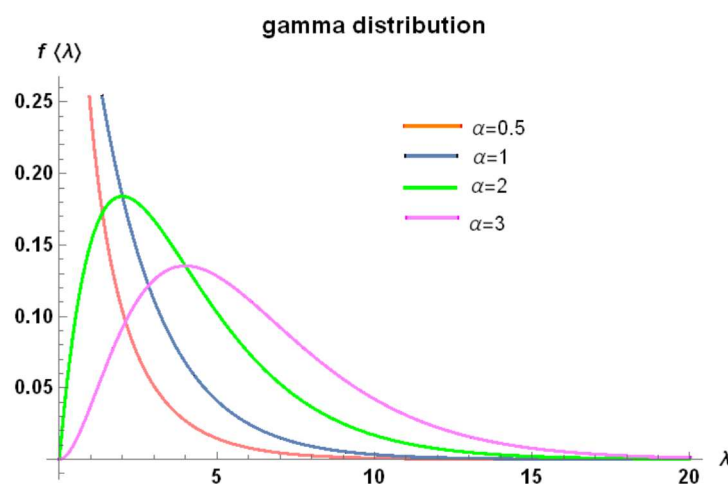

gamma distribution

Figure 5-9   Gamma densities with four shape parameters

B. Glossary [2]

Bayesian inference

Statistical inference involving the use of Bayesian methods. Bayesian inference uses probability distributions to model uncertainty in unknown quantities. Thus, unknown parameters are treated formally as if they were random variables. See also frequentist Inference and statistical inference.

Confidence interval

In the frequentist approach, a 100p% confidence interval has a probability p of containing the true unknown parameter. This is a property of the procedure, not of any one particular interval. Any one interval either does or does not contain the true parameter. However, any random data set leads to a confidence interval, and 100p% of these contain the true parameter. Compare with credible Interval.

Conjugate

A family of prior distributions is conjugate, for data from a specified distribution, if a

prior distribution in the family results in the posterior distribution also being in the family. A prior distribution in the conjugate family is called a conjugate prior. For example, the gamma distributions are conjugate for Poisson data, and the beta distributions are conjugate for binomial data.

Credible interval

In the Bayesian approach, a 100p% credible interval contains 100p% of the Bayesian probability distribution. For example, if A has been estimated by a posterior distribution, the 5th and 95th percentiles of this distribution contain 90% of the probability, so they form a (posterior) 90% credible interval. It is not required to have equal probability in the two tails (5% in this example), although it is very common. For example, the interval bounded by 0 and the 90th percentile would also be a 90% credible interval, a one-sided interval. Bayes credible intervals have the same intuitive purpose as frequentist confidence intervals, but their definitions and interpretations are different.

Cumulative distribution function (c.d.f.)

This function gives the probability that the random variable does not exceed a given value x. For a random variable X, the c.d.f. $F(x) = \Pr(X \leq x)$. If X is discrete, such as a count of events, the c.d.f. is a step function, with a jump at each possible value of X. If X is continuous, such as a duration time, the c.d.f. is continuous. See also **probability density function**. Do not confuse the statistics acronym c.d.f. with the PRA acronym CDF, denoting core damage frequency!

Frequentist inference

Statistical inference that interprets the probability of an event as the long-term relative frequency of occurrence of the event, in many repetitions of an experiment when the event may or may not occur. Unknown parameters are regarded as fixed numbers, not random. See also Bayesian inference and statistical inference.

Initiating event

Any event, either internal or external to the plant, which triggers a sequence of events that challenge plant control and safety systems, whose failure could potentially lead to core damage or large early release.

Interval

The notation (a, b) denotes the interval of all points from a to b. This is enough for all the applications in this report. However, sometimes an additional refinement is added, giving a degree of mathematical correctness that most readers may ignore: The standard notation in mathematics is that (a, b) includes the points between a and b, but not the two end points. In set notation, it is $\{x \mid a < x < b\}$. Square brackets show that the end points are included. Thus, (a, b) includes b but not a, $\{x \mid a < x \leq b)$.

Interval estimate

One way of estimating a parameter is to identify that it falls in some interval (L, U) with a specified degree of certainty, or confidence. The interval (L, U) is referred to as an interval estimate of the parameter. L and U are calculated from the random data. The frequentist interval estimate is referred to as a confidence interval. It does not give a probability statement about the true parameter value. Rather, the interpretation of a $100(1- \alpha)\%$ confidence interval is that, if the random data were drawn many times, $100(1 - \alpha)\%$ of the resulting interval estimates would contain the true value. A Bayesian interval estimate is referred to as a subjective probability interval, or credible interval, and can be interpreted as giving a subjective probability statement about the true parameter value being contained in the interval. Compare with point estimate. See also **confidence Interval**, **credible interval**.

Likelihood

For discrete data, the likelihood is the probability of the observations. For continuous data, the likelihood is the joint density of the observations, which is the product of the densities of the individual observations if the observations are independent. When some of the observations are discrete and some are continuous, the likelihood is the product of the two types. The likelihood is typically treated as a function of the parameters, with the data regarded as fixed.

Maximum likelihood estimator

For data generated from a distribution with one unknown parameter, say a, the maximum likelihood estimate (MLE) of 0 is the parameter value that maximizes the likelihood of the data. It is a function of the data, and is commonly denoted 0. The MILE is a popular frequentist estimator for two reasons. (1) In commonly used models, the MLE is an intuitively natural function of the data. (2) Under certain, commonly valid, conditions, as the number of observations becomes large the MLE is approximately unbiased with approximately the minimum possible variance, and is approximately

normally distributed.

Mean

The mean, μ, of a random variable X is the weighted average of the outcomes, where the weights are the probabilities of the outcomes. More precisely, the mean of X is the expected value E(X), $\sum x_j f(x_j)$ if X is discrete with p.d.f., and $\int xf(x)dx$ if X is continuously distributed with density f.

Periodical inspection reporting system [4]

Periodical inspection reporting system is a system that the persons with the qualifications of building equipment and elevators inspection give periodical tests and then report them to the specific administrative agency of its jurisdiction based on the Building Standard Act Article 1.

Percentile

Consider a continuous distribution with density (p.d.f.) f and cumulative distribution function (c.d.f.) F. The 100qth percentile is the value x such that

F(x) = q, or equivalently $\int_{-\infty}^{x} f(u)\,du = q$

If the distribution is concentrated on the positive line, the lower limit of integration may be replaced by 0. The 100qth percentile is equal to the qth quantile. For example, the 95th percentile equals the 0.95 quantile. If X has a discrete distribution, a percentile may not be unique. The 100qth percentile is defined in this case as x such that $Pr(X \leq x) \geq 100q\%$ and $Pr(X \geq x) \geq 100(1 - q)\%$

Point estimate

An estimate of a parameter in the form of a single number is called a point estimate of the parameter. For example, the mean of a sample of values of a random variable X is a commonly used point estimate of the mean of the distribution. Compare with Interval estimate.

Poisson process

A process in which events (such as failures) occur in a way such that the number of event X in total time t is described by a Poisson distribution.

Posterior distribution

A distribution that quantifies, in a Bayesian way, the belief about a parameter after data have been observed. It reflects both the prior belief and the observed data.

Prior distribution

A distribution that quantifies, in a Bayesian way, the belief about a parameter before any data have been observed.

Probability model

A term for the set of mathematical relationships which are used to define both cumulative distribution functions and either probability distribution functions (discrete case) or probability density functions (continuous case).

Probability density function (p.d.f.)

For a continuous random variable X, the probability density function f satisfies

$$\Pr(a \leq X \leq b) = \int_a^b f(x)dx$$

Properties of the density are

- $f(x) \geq 0$ for all x,

- $\int_{-\infty}^{\infty} f(x)dx = 1$,

- $f(x)\Delta x \approx \Pr(x < X \leq x + \Delta x)$ for small Δx,

The p.d.f. is related to the c.d.f. by $f(x) = F'(x)$, the derivative,

and $F(x) = \int_{-\infty}^{x} f(u)du$ See cumulative distribution function.

Probability distribution function (p.d.f).

For a discrete random variable X, the p.d.f. $f(x) = \Pr(X = x)$

5.5 Reference

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6 Conclusion

The fire at the Browns Ferry Nuclear Plant that revealed shortcomings both in fire protection design at nuclear power plants and the licensee's procedures for responding to a fire has fundamentally changed how the NRC dealt with fire protection at nuclear power plant.

There are many regulatory changes in fire protection. The risk-informed performance-based fire protection approach was also developed as well, which relies on required outcomes (risk insights) rather than specific processes or techniques to achieve those outcomes which required by traditional deterministic fire protection.

Probabilistic Risk Assessment (PRA) used in the risk-informed, performance-based approach is a systematic method for estimating risk by computing real numbers to determine what can go wrong, how likely is it, and what are its consequences. In this report, the parameter estimation for PRA is used for estimating the failure rates of the emergency lighting equipment (one kind of fire safety apparatus) and quantify the uncertainties in the estimates.

Both frequentist and Bayesian approach can be used for the failure rate estimation. However, though the point estimation by the MLE in frequentist approach is simper in form than the Bayes estimates, the Bayesian approach can provide a mechanism for using other information as prior belief for the data that are sparse. In addition, from the analysis results, the credible intervals conducted by Bayes' are always smaller that the confidence intervals conducted by frequentist. In another word, the uncertainties about the failure rate estimates by Bayesian approach are smaller than that by frequentist. Thus, the Bayesian approach works better when estimating parameter (failure rate).

With regard to the Bayesian estimation, if a generic data that can be as a prior existing, the Bayes with informative prior could be a more proper approach for failure rate estimating because of the smaller uncertainty about the estimate. Besides, the posterior mean is always between the prior mean and the MLE if the prior mean exists, so before making an exact calculation of the posterior mean, we could also quickly determine a range that the posterior mean is belong to, which is based on the MLE and prior mean.